FDPE Microeconomic Theory
Spring 2017

Problem Set 1 (Due January 20)

1. Consider strategic form games with two players and two actions available for each player. Let Σ_i denote the set of mixed strategies for each player. Denote by $B_i(\sigma_j) \subset \Sigma_i$ the set of best-responses of player i to player j's strategy σ_j. In other words,

$$B_i(\sigma_j) := \left\{\sigma_i \in \Sigma_i \left| u_i(\sigma_i, \sigma_j) \geq u_i\left(\sigma_i', \sigma_j\right) \text{ for all } \sigma_i' \in \Sigma_i. \right.\right\}$$

   For each of the classic 2x2 games listed in the lecture notes (prisoner's dilemma, stag hunt, battle of sexes, hawk-dove, matching pennies) derive $B_1(\sigma_2)$ for all $\sigma_2 \in \Sigma_2$. Draw a figure of the best-response correspondence in each case. Find all Nash equilibria of each of the games.

2. (Guess the average). Consider the n-player game where all the players announce simultaneously a number in the set $\{1, ..., K\}$ and a price of \$1 is split equally among all the players having the guess closest to $\frac{2}{3}$ of the average of the announced numbers. Find the strategies that are rationalizable (i.e. survive iterated elimination of strictly dominated strategies) and find all Nash equilibria of the game.

3. There are two players with strategy spaces $S_i = \{1, 2, 3\}$, $i = 1, 2$. Each player wants to choose the highest possible number, but both get zero if they pick the same number:

$$u_i(s_i, s_j) = \begin{cases} s_i \text{ if } s_i \neq s_j \\ 0 \text{ if } s_i = s_j \end{cases}, \; i, j = 1, 2, \; i \neq j.$$

   (a) Which strategies are rationalizable? (i.e. survive iterated elimination of strictly dominated strategies)
   (b) Find all (pure and mixed) Nash equilibria of the above game.
   (c) If you have time, think whether you can generalize your answers to a game with strategy spaces $S_i = \{1, ..., N\}$

4. Consider the following two-player game:

	L	R
U	$5,1$	$0,0$
D	$4,4$	$1,5$

   (a) Find all Nash equilibria of the game. What is the best payoff that the players can get in a symmetric equilibrium?

   (b) Suppose that before choosing their actions, the players first toss a coin. After publicly observing the outcome of the coin toss, they choose simultaneously their action. Draw the extensive form game and define available strategies for the players. Find a Nash equilibrium that gives both players a higher payoff than the symmetric equilibrium in a).

   (c) Suppose that there is a mediator that can make a recommendation separately for each player. Suppose that the mediator makes recommendation (U, L), (D, L) or (D, R), each with probability $1/3$. Each player only observes her own action choice recommendation (so that, e.g., player one upon seeing recommendation D does not know whether the recommended profile is (D, L) or (D, R)). Does any of the players have an incentive to deviate from the recommended action? What is the expected payoff under this scheme?

   The solution concept that this excercise demonstrates is called correlated equilibrium (see Myerson, Osborne-Rubinstein, or Fudenberg-Tirole for full discussion of the concept).

5. Consider a simple model of R&D race. Two firms choose simultaneously how much money to invest and the winner is the firm who invests more. The winner gets a prize worth 1 million Euros. If both firms invest the same amount, then each firm wins with probability $1/2$. The loser gets no prize. Formulate this situation as a strategic form game and analyze it (hint: look for a symmetric Nash equilibrium in mixed

strategies i.e. a probability density function for investment level that keeps players indifferent between different amounts).