FDPE Microeconomics 3
Spring 2017

Problem Set 2 (Due January 27)

1. Consider a zero-sum game, where the sets of pure strategies are $S_i = \{1, ..., K\}$, $i = 1, 2$, and payoffs are

$$u_1(s_1, s_2) = -u_2(s_1, s_2) = \begin{cases} 1 \text{ if } s_1 = s_2 \\ -1 \text{ if } s_1 \neq s_2 \end{cases}.$$

   (a) Compute maxmin payoffs when only pure strategies are allowed:

$$\max_{s_1 \in S_1} \min_{s_2 \in S_2} u_1(s_1, s_2) \text{ and } \max_{s_2 \in S_2} \min_{s_1 \in S_1} u_2(s_1, s_2).$$

   (b) Compute

$$\min_{s_2 \in S_2} \max_{s_1 \in S_1} u_1(s_1, s_2) \text{ and } \min_{s_1 \in S_1} \max_{s_2 \in S_2} u_2(s_1, s_2).$$

   (c) Now, allow mixed strategies and compute

$$\max_{\sigma_1 \in \Delta(S_1)} \min_{\sigma_2 \in \Delta(S_2)} u_1(\sigma_1, \sigma_2) \text{ and } \max_{\sigma_2 \in \Delta(S_2)} \min_{\sigma_1 \in \Delta(S_1)} u_2(\sigma_1, \sigma_2).$$

   (d) Compute

$$\min_{\sigma_2 \in \Delta(S_2)} \max_{\sigma_1 \in \Delta(S_1)} u_1(\sigma_1, \sigma_2) \text{ and } \min_{\sigma_1 \in \Delta(S_1)} \max_{\sigma_2 \in \Delta(S_2)} u_2(\sigma_1, \sigma_2).$$

   (e) Find Nash equilibria in pure strategies.

   (f) Find Nash equilibria in mixed strategies.

2. (War of attrition) Two players are fighting for a prize whose current value at any time $t = 0, 1, 2, ...$ is $v > 1$. Fighting costs 1 unit per period. The game ends as soon as one of the players stops fighting. If one player stops fighting in period t, he gets no prize and incurs no more costs, while his opponent wins the prize without incurring a fighting cost. If both players stop fighting at the same period, then

neither of them gets the prize. The players discount their costs and payoffs with discount factor δ per period.

This is a multi-stage game with observed actions, where the action set for each player in period t is $A_i(t) = \{0, 1\}$, where 0 means continue fighting and 1 means stop. A pure strategy s_i is a mapping $s_i : \{0, 1, ...\} \rightarrow A_i(t)$ such that $s_i(t)$ descibes the action that a player takes in period t if no player has stopped the game in periods $0, ..., t-1$. A behavior strategy $b_i(t)$ defines a probability of stopping in period t if no player has yet stopped.

(a) Consider a strategy profile $s_1(t) = 1$ for all t and $s_2(t) = 0$ for all t. Is this a Nash equilibrium?

(b) Find a stationary symmetric Nash equilibrium, where both players stop with the same constant probability in each period.

(c) Are the strategy profiles considered above sub-game perfect equilibria?

(d) Can you think of other strategy profiles that would constitute a sub-game perfect equilibrium?

3. Consider a two-player stopping game with a finite time horizon $t = 0, 1, 2, ..., T$. At each period, both players choose simultaneously whether to stop or continue. The game ends as soon as one of the players stop. The payoffs are given by $u_1(t) = u_2(t) = t$, if the game ends at period t. If no player ever stops, both players get zero.

(a) Find all Nash equilibria. Are there sub-game perfect equilibria?

(b) Let the time horizon be infinite, that is, $t = 0, 1, ...$ The same questions as in a).

(c) The game is otherwise as in b), but at every period where both players choose "continue", the game ends with exogenous probability $p > 0$. If that happens before any of the players choose to stop, then both players get zero. Find all Nash equilibria and sub-game perfect equilibria of the game.

4. Consider the simple card game discussed in the lecture notes: Players 1 and 2 put one dollar each in a pot. Then, player 1 draws a card from a stack, observes privately the card, and decieds wheter to "raise" or "fold". In case of "fold", game ends and player 1 gets the money if the card is red, while player 2 gets the money if black. In case of "raise", player 1 adds another dollar in the pot, and player 2 must decide whether to "meet" or "pass". In case of "pass", game ends and player 1 takes the money in the pot. In case of "meet", player 2 adds another dollar in the pot, and player 1 shows the card. Player 1 takes the money if the card is red, while player 2 takes the money if black.

   (a) Formulate the card game as an extensive form game.

   (b) Represent the game in strategic form and find the unique mixed strategy Nash equilibrium of the game.

   (c) Write the corresponding equilibrium using behavior strategies.

   (d) Derive a belief system (probabilities for nodes within each information set) that is consistent with the equilibrium strategies (i.e., derived using Bayesian rule).

   (e) Check that the equilibrium strategies are sequentially rational given the belief system that you derived in d).

5. An entrant firm (player 1) decides whether to enter an industry with an incumbent firm (player 2). Entry costs $c = 1$. If there is no entry, then player one gets payoff of 0 and player 2 gets payoff of 3. If there is entry, then the firms decide simultaneously whether to fight or cooperate, with payoffs given in the matrix below (so that the total payoff of player one is the payoff given in the matrix minus her entry cost):

	Fight	*Cooperate*
Fight	$-1, 0$	$0, -1$
Cooperate	$0, 0$	$2, 2$

   (a) Define the extensive form game.

(b) Find all Nash equilibria.

(c) Find all Sub-game perfect Nash equilibria.

(d) Find all weak perfect Bayesian equilibria.

(e) Find all sequential equilibria.

6. Two players are contributing to a public good over time. Player 1 contributes in odd periods and player 2 in even periods. If player i contributes in period t amount z_{it}, she bears individually cost $c_i(z_{it}) = z_{it}$. All past contributions are irreversible and publicly observable. Once the toal cumulative contribution exceeds a threshold $\overline{z}$, both players get a one time payoff π and the game is over. The players maximize their payoff net of their individual cost of providing the public good. Assume that $\pi < \overline{z} < 2\pi$.

   (a) For the case where $t \in \{1, 2\}$, find the subgame perfect equilibria of the game. Are there other Nash equilibria?

   (b) The same questions with $t \in \{1, ..., T\}$.

   (c) Assume that the time horizon is infinite. What kind of sub-game perfect equilibria can you find?