We ended the last problem set with the game which exposed the drawbacks of PBE in the games of incomplete information. Problem 1 below presents some games where PBE does not perform well, and introduces sequential equilibrium (SE) as a remedy to the exposed problems.

Problem 1 (Sequential equilibrium)

a) In the game of Figure 1, Nature chooses L with probability $\frac{3}{4}$. What are SE of the game? Compare them to PBE found in problem set 2: does SE make more appealing predictions on the outcome of the game?

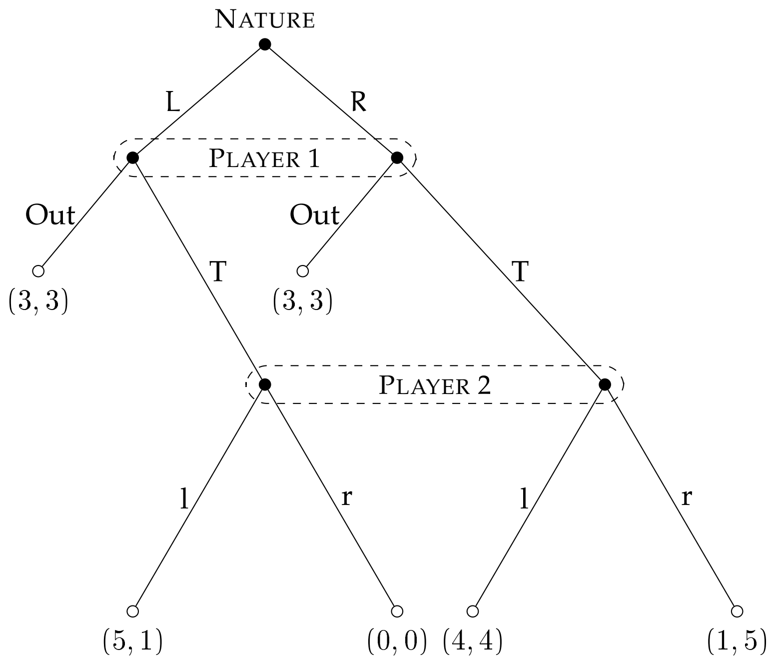

Figure 1: SPE supported by inconsistent beliefs

b) Show that the set of SPE is a proper subset of the PBE in the game of Figure 2. What are SE of the game?

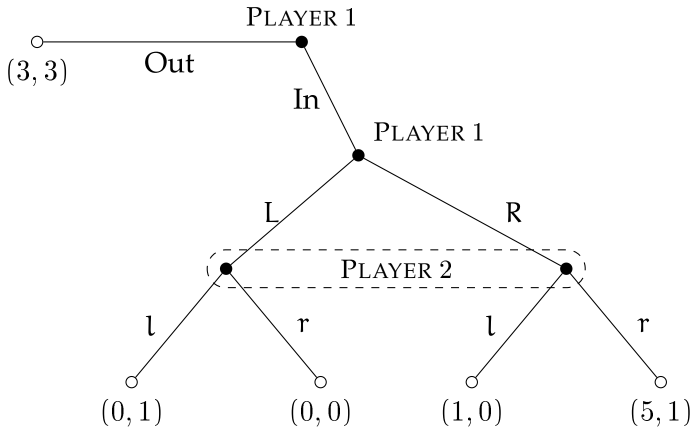

Figure 2: Pathological PBE

In lecture notes 3, we analyzed SPE of sequential bargaining game with alternating offer protocol - i.e., players took turn making offers. In problem 2, we generalize the protocol but focus on a two-period model - and show that the main insights of the model do not change.

Problem 2 (Random proposer protocol) *Players* 1 *and* 2 *want to divide a dollar and they have two periods to reach an agreement. Players are risk-neutral, and if the agreement is not reached by the end of period* 2*, Nature sets the dollar on fire. Nature chooses player* 1 *to make a proposal on a division of the dollar in period* $t \in \{1, 2\}$ *with probability* π*, and with complementary probability it is player* 2*, who gets to make a proposal in period* t*. That is, in period* 1 *the player recognized as a proposer by Nature suggests a division of the dollar* $(x^1, 1-x^1)$*, and the other player can either accept or refuse this proposal. If the offer is accepted, the game ends with payoffs* $(x^1, 1-x^1)$*. If the offer is refused, the game moves to period* 2*, where Nature chooses a proposer again [she chooses player* 1 *with probability* π*, and player* 2 *with probability* $(1-\pi)$*], and the recognized player proposes a division* $(x^2, 1-x^2)$*. If the offer is accepted, the game ends with payoffs* $(\delta x^2, \delta(1-x^2))$*. If the offer is rejected, the game ends with payoffs* $(0, 0)$*. What is the unique* SPE *of the game?*

Problem 3 *A popular strategy suggestion for playing a repeated prisoner's dilemma is called tit-for-tat. In that strategy, both players start by cooperating* (C, C) *and in any period* t*, they replicate the action of their opponent in period* $t-1$*. Consider the infinitely repeated game where both players discount future* *with discount factor $\delta < 1$. The stage-game payoffs are:*

	C	D
C	$3, 3$	$0, 4$
D	$4, 0$	$1, 1$

Write down a formal definition for the tit-for-tat strategy. Is the strategy profile where both players play tit-for-tat a Nash equilibrium?
Is it a sub-game perfect Nash equilibrium?

Problem 4 *Consider a two-stage game with observed actions, where in the first stage players choose simultaneously U1 or D1 (player 1) and L1 or R1 (player 2), and in the second stage players choose simultaneously U2 or D2 (player 1) and L2 or R2 (player 2). The payoffs of the stage games are shown in the tables below:*

First stage:

	L1	R1
U1	$2, 2$	$-1, 3$
D1	$3, -1$	$0, 0$

Second stage:

	L2	R2
U2	$6, 4$	$3, 3$
D2	$3, 3$	$4, 6$

The players maximize the sum of their stage-game payoffs.

a) Find the subgame-perfect equilibria of this game.

b) Suppose that the players can jointly observe the outcome y_1 of a public randomizing device before choosing their first-stage actions, where y_1 is drawn from uniform distribution on the unit interval. Find the set of subgame-perfect equilibria, and compare the set of possible payoffs against the possible payoffs in a).

c) Suppose that the players jointly observe y_1 at the beginning of stage 1 and y_2 at the beginning of stage 2, where y_1 and y_2 are independent draws from a uniform distribution on a unit interval. Again, find the sub-game perfect equilibriua and possible payoffs.

Problem 5 (Folk Theorem) *Consider an infinitely repeated game with a stage* *game given in the following matrix:*

	L	R
U	$5, 0$	$0, 1$
M	$3, 0$	$3, 3$
D	$0, -1$	$0, -1$

Players have a common discount factor.

a) Find the minmax payoffs for each of the players.

b) Characterize the set of feasible payoff vectors of the stage game (Assume that a public randomization device is available).

c) What is the set of normalized payoff vectors for the repeated game, such that each element in the set is a subgame perfect equilibrium payoff vector for some value of the discount factor?

d) Can you construct some subgame perfect equilibrium strategies leading to the constant play of (U, L) in the equilibrium path?

e) Let's change the game so that payoffs for (D, L) and (D, R) are $(0, 0)$. Can there now be an equilibrium with a constant play of (U, L)?

Problem 6 *Consider a model where two sellers sell an identical good to a single consumer (without storage possibilities) over an infinite horizon. The firms compete by setting prices simultaneously at the beginning of each period and the consumer chooses which of the prices to accept at the end of the stage. The consumer has unit demand in each period, i.e. she is willing to pay up to v in each period to buy one unit. Additional units are worthless to the buyer. Assume that the good can be produced at marginal cost c.*

a) Suppose that the buyer is myopic, i.e. she has a discount factor $\delta^C = 0$ whereas the firms are patient and have a discount factor $0 < \delta^F < 1$. What is the smallest δ^F that is compatible with collusive pricing in the market in subgame perfect equilibrium? I.e. for what δ^F is it possible to set prices $p_{it} = v$ for all i and all t *on the equilibrium* path? What is the punishment path supporting this? (Hint: what are the strategies of the players?)

b) Suppose next that all players, i.e. the sellers as well as the buyer have the same discount factor δ. Can you find an equilibrium where collusion is possible at a δ below that found in the previous part? (Hint: try to constract strategies for the sellers that reward the buyer for not falling for a price cut of the competitor)