Empirical research on discrete choice game theory models of entry: An illustration

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Abstract

We discuss the empirical implementation of discrete game theoretic models of firm entry. After presenting a simple model of entry that underlies much of existing empirical analysis, we discuss the major problems that must be tackled when empirically implementing theoretical models, and econometric methods used in the literature. Finally, we present results from a reduced form estimation of a sequential move entry game, using data from the UK hamburger market. © 2000 Elsevier Science B.V. All rights reserved.

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1. Introduction

The last decade has witnessed important advances both in our ability to translate game theoretic models involving discrete choices into econometric models, and in developing relevant estimation methods. Several contributions combine these advances,\textsuperscript{1} promising a better understanding of the issues, and

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\textsuperscript{1}Examples are Berry (1992), Davis (1999) and Mazzeo (1999). For a general survey of findings on entry, see e.g. Geroski (1995).
See Reiss (1996) for a useful discussion of modelling approaches. The objective of this paper is to outline some of the key ideas involved in game theoretic models of entry, then to present some results using data on the UK fast food duopoly.

The remainder of the paper is structured as follows. In Section 2, we discuss a static two-firm entry model, the various issues that arise in the construction of an econometric model, the modelling choices made thus far in the literature, and the econometric methods used. In Section 3 we present an application to the UK fast food market, and some reduced form results. The fourth and last section concludes.

2. Estimation of discrete-choice game theoretic models of entry

Consider an extremely simple two-stage model of entry, where two identical firms producing homogeneous goods decide in the first period whether or not to enter a market. In the second period, conditional on entry, they compete in quantities, for example (what is important is that the firms know the form of second stage competition). The literature concentrates on pure strategy equilibria. Following entry, firms earn profit $\Pi_i$ ($i \in \{\text{Mono, Duo}\}$, Mono = monopoly, Duo = duopoly), but in order to enter have to pay a fixed cost of entry, denoted $F > 0$. Thus a firm enters if

$$E[\Pi_i] - F \geq 0.$$ (1)

Assuming entry by at least one firm is profitable (but suppressing the determinant of profit for the moment), we expect to see a monopoly structure if

$$\Pi^{\text{Mono}} - F \geq 0 \quad \text{and} \quad \Pi^{\text{Duo}} - F < 0,$$ (2)

whereas a duopoly emerges if

$$\Pi^{\text{Duo}} - F \geq 0.$$ (3)

All existing structural empirical work on entry relies on some kind of variant of this simple model. However, there are at least three significant decisions to be made in doing so. The first is that when estimating a simultaneous entry model along these lines, a natural inclination is to use a system of two (or more generally, $N$, where $N$ is the number of potential entrants) discrete choice (e.g. probit) equations. This is however not straightforward. Firstly, such an econometric model is not identified, as each (probit) equation would have rival $k$’s entry, a dichotomous variable, as an explanatory variable in the equation for $i$’s entry. As discussed by Heckman (1978) and Bresnahan and Reiss (hereafter, BR) (1991a), such a model is identified if and only if the system is recursive. In the

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2 See Reiss (1996) for a useful discussion of modelling approaches.
current context, maintaining this assumption would imply that firm $i$’s profits are not affected by firm $k$’s entry (or vice versa), which is infeasible. Secondly, the model does not provide a one-to-one mapping from Nash equilibria of the game to observed outcomes. The culprits for this are the monopoly outcomes, where we know that only one or the other firm will enter in equilibrium, but do not know which firm enters and which does not.

There have been two main ways out of this conundrum. Bresnahan and Reiss (1990, 1991a,b) noted that the model provides a one-to-one mapping from the equilibrium to the number of firms. That is, if
\[
\Pi^{N+1}(\cdot) - F < 0 \quad \text{and} \quad \Pi^N(\cdot) - F > 0,
\] then an $N$th firm finds it profitable to enter in equilibrium, but not an $(N + 1)$th. Therefore, assuming that the fixed costs are normally distributed, (4) suggests that one should estimate an ordered probit, as BR do.\footnote{This approach has been extended by Davis (1999) to allow for multi-plant firms. He shows that under certain assumptions (including product homogeneity), the model generates a unique prediction of the number of stores in a market, even though the identity of stores is indeterminate.}

The other solution is to change the structure of the game so as to obtain unique pure strategy equilibria for all values of the parameters. One such change is to assume that firms make their entry decisions sequentially instead of simultaneously.\footnote{Again, see Reiss (1996). This assumption is utilised by Toivanen and Waterson (1999) and Mazzeo (1999). In his empirical application Berry (1992) also imposes an order of entry.} Naturally, imposing an assumption about the order of entry requires good knowledge of the industry, including the identity of the potential entrants, as this may have a large impact on the results (see Toivanen and Waterson (hereafter TW), 1999).

The second important decision concerns how to allow for firm heterogeneity. BR (1990) discuss different ways of introducing this into determinants of individual firm profit via the error term. They specify the deterministic part of the profit function to be
\[
\Pi = SV - F,
\] where $S$ is a function that determines (the firm’s share of) market size, and $V$ a function that determines variable profits, the latter being a decreasing function of the number of firms. Subsequent studies have adopted this convenient structure for the deterministic element. BR then estimate a model that allows for $V$ to be determined with error. Also, and perhaps more importantly, they raise the issue of unobserved firm heterogeneity. It is quite plausible that there are important firm-specific characteristics that affect entry decisions yet are not observed by the econometrician. BR therefore estimate (BR, 1990) a model where they allow that all those firms entering their markets ‘first’ have
the same unobserved component of fixed costs. Though this is an important step in the right direction, there are two rather stringent constraints involved. For one thing, it is not at all clear that all ‘first’ or ‘second’ etc. entrants should have the same cost structure. Secondly, one could well argue that there are important (unobserved) differences in the profits firms earn in a market, due for example to product differentiation.

Berry’s work (1992) relaxes the first of these constraints relating to fixed costs. Two more recent papers allow for heterogenous profits. Mazzeo (1999) uses data on motels on US highway exits to estimate a model where there are high- and low-quality motels. The profit functions differ between the groups, but are homogenous within groups. TW use data on the UK fast food (hamburger) duopoly. Their approach differs from those previously employed in that they estimate firm-specific entry functions. The benefit is naturally that one can allow for firm-specific coefficients in (5) in addition to market (and firm-) specific unobserved heterogeneity that may stem from either profits or fixed costs.

Finally, the researcher has to decide how to treat existing firms in the market as opposed to de novo entrants. The existing game-theoretic work, apart from Scott-Morton (1999) and TW, assumes that continuation of existing operations is identical to new entry. One could therefore view these papers as explaining existing market structure rather than entry. Clearly, whether it is desirable to view all players as entering every period, or whether incumbent operators’ decisions are different, will depend on the nature of the market in question.

3. An application

We now turn briefly to an application. In Toivanen and Waterson (1999), we study the UK hamburger duopoly over the period 1991–1995. The two firms in question (McDonalds (McD), and Burger King (BK)) have aggressive expansion policies that result in a large number of new outlets. We observe that these two firms have a combined market share of around 60%, and that the third firm (with a market share of 18%) is effectively barred from the relevant market for most of the observation period. We then argue that all other firms are too small to be considered ‘strategic players’, and that the effects of these firms, if any, on the established duopolists’ entry behaviour can be controlled for by market-specific controls. We also argue, based on the nature of the entry patterns observed over time in the data, that McD is likely to be a leader in the market

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5 Chevalier (1995) investigates de novo entry, but her econometric model is not based on a game theoretic approach.
6 Thus in BR, for example, it is not actually entry which is being examined, rather a snapshot of active players at a point in time.
and BK its follower, and therefore assume in the application that these firms make entry decisions sequentially. The data cover 453 separate geographical markets, resulting in 2265 observations.

Here we report estimates from an empirical model with the following reduced form second-stage profit function developed from Berry (1992):

\[ \Pi_{ijt} = X_{ijt} \beta_i + g_i(B_{ijt}, M_{ijt}, \theta_i) + \nu_{ijt}. \]  

We are concerned very much with the identities of the players. Thus, subscript \( i \) denotes (\( i \in \{ M, B \} \), where \( M \) stands for McD, \( B \) for BK), \( j \) the market and \( t \) the time period; the vector \( X_{ijt} \) includes market and (possibly) firm specific variables; \( g_i(\cdot) \) is a function of existing own and rival outlets in market \( j \) (see also Mazzeo, 1999); and \( \beta_i \) and \( \theta_i \) are (firm-specific) parameter vectors to be estimated. We denote the number of outlets of firm \( i \) in market \( j \) in period \( t \) by \( i_{jt} \). The error term \( \nu_{ijt} \) captures the effects of events not observed in the measured data.

Because our firms are multi-plant duopolists, we examine the profit difference from opening the \( n \)th store in market \( j \) as compared to continuing to operate the existing \( n - 1 \) stores rather than using (6) directly. We specify in this paper (TW use more general functional forms) that

\[
\begin{align*}
g_B(\cdot) &= \theta_{B1} M_{jt} + \theta_{B2} B_{jt} + \theta_{B3} B_{jt} M_{jt}, \quad (7a) \\
g_M(\cdot) &= \theta_{M1} B_{jt} + \theta_{M2} M_{jt} + \theta_{M3} M_{jt} B_{jt}. \quad (7b)
\end{align*}
\]

Our aim is to estimate whether or not entry occurs. For the first outlet, Eqs. (6) and (7) characterise the expected profits from entering. The constant term will (along with \( \nu_{ijt} \)) capture the fixed costs of entry and we may assume that if expected profits exceed fixed costs then entry will occur. However, for a firm with one or more existing outlets, which is contemplating entry, it is the increase in expected profits minus the fixed costs of entry that matter. For the follower (BK), this results in the following equation:

\[ \Pi_{Bjt}(B) - \Pi_{Bjt}(B - 1) = \theta_{B1} + \theta_{B3} M_{jt} + F_B + \zeta_{Bjt}. \]  

The first two terms come from (7) and the last two give the fixed cost of entry into market \( j \) by BK in period \( t \). The last term is an i.i.d error term. Note that this formulation does not allow us to identify separately \( \theta_{B1} \) and \( F_B \). For the leader (McD), the equation has to be amended to allow for the fact that the follower will react to the leader’s decision (not) to enter:

\[
\begin{align*}
\Pi_{Mjt}(M) - \Pi_{Mjt}(M - 1) &= \theta_{M1} + \theta_{M2} [B_{jt}(M_{jt}) - B_{jt}((M - 1)_{jt})] \\
&\quad + \theta_{M3} [M_{jt} B_{jt}(M_{jt}) - (M - 1)_{jt} B_{jt}((M - 1)_{jt})] + F_M + \zeta_{Mjt}, \quad (8b)
\end{align*}
\]
where $B_{jt}(M_{jt})$ denotes the number of BK outlets as a function of the number of McD outlets. Again, the constant and fixed costs cannot be separately identified. We use (6)–(8) in constructing the likelihood function.

A problem we need to tackle is that there may be (firm and) market specific profits (entry costs) that are unobserved by the econometrician. If so, firms will enter markets that have high unobserved profits (low unobserved entry costs) with a higher probability than observationally equivalent markets with lower unobserved profits (high entry costs). This in turn would lead to an upward bias in our market structure coefficients. To allow for such unobservables, we estimated a model where the error term (a combination of $\xi_{ijt}$ and $v_{ijt}$ for each firm) took the form

$$
\eta_{ijt} = \rho \xi_{ijt} (1 + \phi_{iM} M_{jt} + \phi_{iB} B_{jt}) + \epsilon_{ijt} \sqrt{1 - \rho^2}.
$$

In other words, we imposed the standard normalization that $\text{var}(\eta_{ijt}) = 1$ (when $\phi_{ik} = 0$), and allowed the (firm and) market specific error component $\xi_{ij}$ to be correlated with the number of existing own and rival outlets. In (9), $\rho$ is the proportion of total variance due to the market specific error term $\xi_{ij}$.

We resorted to a simulation estimator (see e.g. Berry, 1992) in order to estimate a model with this error structure, and to solve the endogeneity problem involved in estimating the leader’s entry decision. To simulate the equi-correlated error term, we took $P$ ($P = 30$) simulation draws. To simulate BK entry we took $R$ ($R = 10$) random draws from a standard normal distribution, and calculated BK’s response to McD (not) entering a given market in that period (using the estimated BK follower decision rule). These expected values are then used in the estimation of McD’s decision. Identification of the model was achieved through exclusion and functional form restrictions. Our estimation of the models with market-specific unobserved profits that are correlated with the number of own and rival outlets yielded an estimate for $\rho$ of 0.000471 (s.e. 0.274612) and 0.000988 (s.e. 0.426104) for McD and BK, respectively. All the $\phi_{ik}$’s were very imprecisely measured, with $p$-values of the order of 0.9. Therefore, we cannot reject the Null of no unobserved market specific profits for either firm, and hence the results reported below come from estimations that assume no unobserved profits.

Column 1 in Table 1 gives the results for BK. As can be seen, the probability of BK entry is positively affected by POPulation and average WAGEs, and negatively by the proportion of PENSIONers. The geographic AREA of the market and the proportion of under-16 years old (YOUTH) obtain imprecisely estimated coefficients. Turning to the market structure estimates, the joint effect of $\theta_{B1}$ and $F_B$ is negative ($F_B$ most likely dominates). We find that rival presence has a positive impact on BK entry, through both the linear and interaction terms (coefficient values 0.240 and 0.170, respectively).

McD’s entry (column 2) is positively affected by the POPulation variable. The other market characteristic variables carry imprecisely estimated coefficients.
Table 1
Estimation results

<table>
<thead>
<tr>
<th>Variable/parameter</th>
<th>BK</th>
<th>McD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta_0 ) (constant)</td>
<td>(-2.103)</td>
<td>(-4.676)</td>
</tr>
<tr>
<td></td>
<td>((1.498))</td>
<td>((1.891))</td>
</tr>
<tr>
<td>( \beta_1 ) (pop)</td>
<td>(0.005)</td>
<td>(0.012)</td>
</tr>
<tr>
<td></td>
<td>((0.001))</td>
<td>((0.003))</td>
</tr>
<tr>
<td>( \beta_2 ) (area)</td>
<td>(-0.191)</td>
<td>(-0.321)</td>
</tr>
<tr>
<td></td>
<td>((0.114))</td>
<td>((0.220))</td>
</tr>
<tr>
<td>( \beta_3 ) (youth)</td>
<td>(-0.092)</td>
<td>(0.060)</td>
</tr>
<tr>
<td></td>
<td>((0.061))</td>
<td>((0.090))</td>
</tr>
<tr>
<td>( \beta_4 ) (pension)</td>
<td>(-0.063)</td>
<td>(0.003)</td>
</tr>
<tr>
<td></td>
<td>((0.029))</td>
<td>((0.028))</td>
</tr>
<tr>
<td>( \beta_5 ) (wage)</td>
<td>(0.169)</td>
<td>(0.101)</td>
</tr>
<tr>
<td></td>
<td>((0.057))</td>
<td>((0.070))</td>
</tr>
<tr>
<td>( \theta_1 + F ) ((m_H), fixed entry cost)</td>
<td>(-1.579)</td>
<td>(-1.223)</td>
</tr>
<tr>
<td></td>
<td>((0.158))</td>
<td>((0.095))</td>
</tr>
<tr>
<td>( \theta_2 ) ((n_H))</td>
<td>(0.170)</td>
<td>(-0.048)</td>
</tr>
<tr>
<td></td>
<td>((0.077))</td>
<td>((0.140))</td>
</tr>
<tr>
<td>( \theta_3 ) ((n_Hm_H))</td>
<td>(0.240)</td>
<td>(0.074)</td>
</tr>
<tr>
<td></td>
<td>((0.034))</td>
<td>((0.035))</td>
</tr>
<tr>
<td>( \log L )</td>
<td>(-436.025)</td>
<td>(-608.272)</td>
</tr>
<tr>
<td>LR</td>
<td>(200.150) (15)</td>
<td>(93.456) (12)</td>
</tr>
</tbody>
</table>

Notes: Dependent variable is entry at time \(t\) (or not). There are 2265 observations. Standard errors are in parentheses. Both equations include year dummies; BK estimation includes in addition a dummy for London, a dummy for (14 cases) missing BK opening dates (see TW), and a dummy for missing wage data (see TW). These dummies were insignificant in the McD estimation and were therefore dropped. LR is a likelihood ratio test of joint significance of all R.H.S. variables (d.f.).

The joint effect of \(\theta_{M1}\) and \(F_M\) is also negative and significant. The interaction term obtains a positive and significant coefficient (value 0.07). These results demonstrate the important finding that, for both firms, rival presence facilitates entry. Moreover, this finding is not due to an upward bias caused by unobserved profits or fixed costs of entry.

4. Conclusions

We have attempted to illustrate how empirical work based on discrete game-theoretic models has evolved in the last decade, in part through the medium of our example. The seminal papers of Bresnahan and Reiss largely set the agenda. The usefulness of simulation estimators, following Berry (1992), removed one obstacle from further development, and all later work uses simulation estimators to some extent to solve the technical problems of potentially
endogenous entry. One clear obstacle to swift progress is the availability of good data, since most recent analyses are based on data sets constructed and collected by the researchers themselves. However, the most important task ahead is to move from using static models of entry, and allow for dynamics.\(^7\)

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**References**


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\(^7\) Ericson and Pakes (1995) allow for entry in a very general model of industry evolution.