**Numerical example to Affiliated Common Value Auctions with Costly Entry by Pauli Murto and Juuso Välimäki**

We present here a numerical example to illustrate the properties of the informal auctions. This example is based on parameter values where case (5) of the paper holds.

Fix the following parameter values: $v(0) = 1$, $v(1) = 2$, $\alpha_{0,l} = \alpha_{1,h} = 0.75$, $\alpha_{0,h} = \alpha_{1,l} = 0.25$, $q = 0.5$. In what follows, we let the entry cost parameter $c$ vary. Proposition 1 now implies that the socially optimal entry profile has both low and high types entering with a strictly positive rate as long as $c < \bar{c} = 0.707$. Figure 1 shows the socially optimal entry rates $\pi_h^{opt}$ and $\pi_l^{opt}$ as functions of $c$.

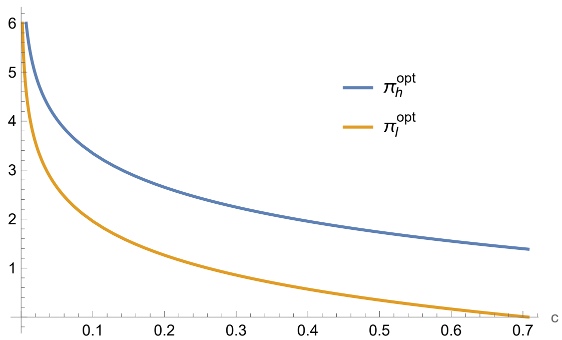


Figure 1: Socially optimal entry rates as a function of $c$.

With our parameters, we have

$$\frac{\alpha_{0,l}}{\alpha_{1,l}} - \frac{v(1)}{v(0)} = 1 > 0.$$

By Proposition 4, this implies that entry is efficient in the informal auction as long as $c$ is above some threshold $\tilde{c}$. The full equilibrium is constructed in the proof of Proposition 2 for all different cases. Following that proof, we note that we have an equilibrium with efficient entry as long as

$$\frac{1 - e^{-\alpha_{0,l}\pi_l^{opt}}}{1 - e^{-\alpha_{1,l}\pi_l^{opt}}} - \frac{v(1)}{v(0)} > 0.$$

(From equation (20) of the paper). To find numerically the critical value for $\tilde{c}$, Figure 2 draws the expression $\frac{1-e^{-\alpha_{0,l}\pi_l^{opt}}}{1-e^{-\alpha_{1,l}\pi_l^{opt}}} - \frac{v(1)}{v(0)}$ as a function of parameter $c$. We see here that this expression is positive for $c > 0.103$. We conclude that the informal auction has efficient entry as long as $c > \tilde{c} = 0.103$. As long as $c < \bar{c} = 0.707$, this involves strictly positive entry by both types of entrants.

In contrast, the formal auction has inefficient entry whenever $c < \bar{c}$ (see Proposition 3 of the paper). Therefore, the informal auction gives strictly superior revenue whenever the entry cost satisfies $c \in (0.103, 0.707)$.

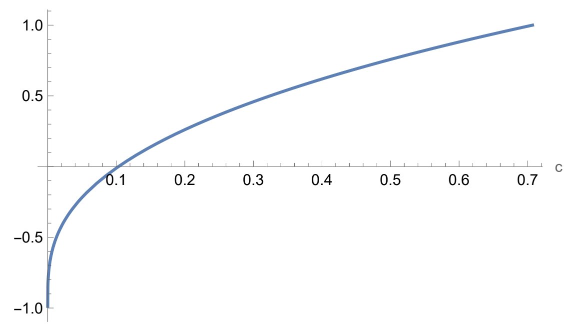


Figure 2: Condition for efficient entry as a function of $c$ (Positive value indicates efficient entry)

We next illustrate the equilibrium bidding distributions for various different values of $c$. We start with a relatively large value $c = 0.5$. With this value of $c$, efficient entry is given by $\pi_h = 1.733, \pi_l = 0.347$. Equation (20) of the paper holds, and we can directly compute the upper bound of the bidding support of the high types as $p^{\max} = q_h v(1) + (1 - q_h) v(0) - c = 1.2$. We can also numerically compute $\overrightarrow{p} = 0.239$ (defined implicitly in the proof of Proposition 2) and $\tilde{p} = 0.5$ (defined explicitly in the proof of Lemma 3). Since $\overrightarrow{p} < \tilde{p}$, we are in the Case 1 of the proof of Proposition 2, that is, the bidding equilibrium consists of two connected supports that contain 0 (see the first schematic figure in the proof of Proposition 2). Figure 3 below shows the bidding distributions of the high and low types. Figure 4 shows the value functions $U_\theta$, $\theta = l, h$.

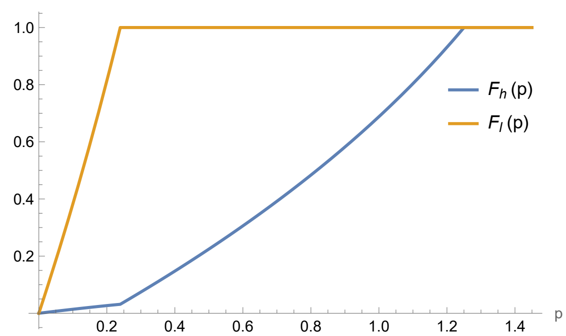


Figure 3: Bidding distributions for c=0.5

As we reduce $c$, the nature of the bidding equilibrium will change at some point. Taking $c = 0.2$, equation (20) still holds and we have efficient entry at $\pi_h = 2.649, \pi_l = 1.263$. We have $p^{\max} = 1.55$, $\overrightarrow{p} = 0.619$, and $\tilde{p} = 0.5$. Since in this case $\overrightarrow{p} > \tilde{p}$, we are in the Case 2 and so the bidding support of the high types now consists of two disconnected intervals (as in the second

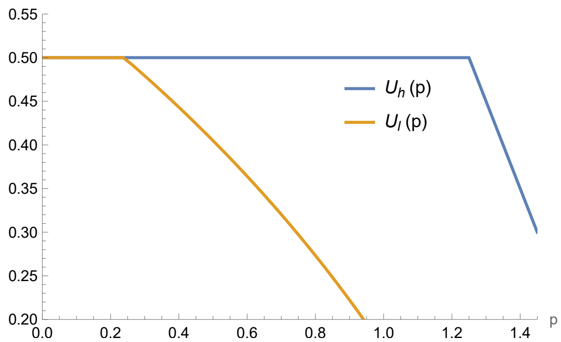


Figure 4: Value of bidding $p$ for c=0.5

schematic figure of the proof of Proposition 2). We can now compute numerically $\overleftarrow{p} = 0.349$. Figure 5 shows the bidding distributions and Figure 6 shows the value functions in this case. Note the gap $(\overleftarrow{p}, \overrightarrow{p}) = (0.349, 0.619)$ in the high type bidding support.

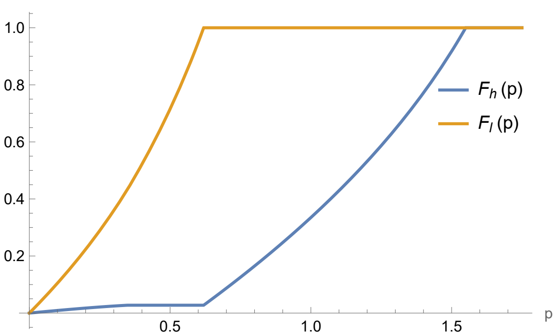


Figure 5: Bidding distributions for c=0.2

Finally, reducing $c$ below $\tilde{c}$ will make overlapping bidding equilibria infeasible. Let $c = 0.01$. In this case, we must have a bidding equilibrium where the supports of the two types do not overlap. Since the high type support no longer contains 0, this is not consistent with efficient entry. Using equations (22) and (23) of the paper we can now solve for the distorted entry rate of the high type: $\pi_h = 6.202 > \pi_h^{opt} = 5.645$. The low type entry rate is correspondingly distorted downwards: $\pi_l = \pi_l^*(\pi_h) = 3.951 < \pi_l^{opt} = 4.259$. Figure 7 shows the bidding distributions and Figure 8 shows the value functions in this case.

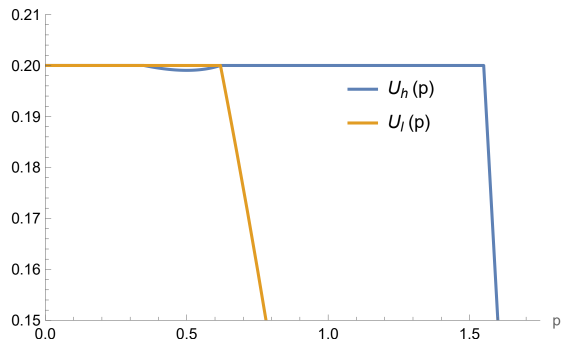


Figure 6: Value of bidding $p$ for c=0.2

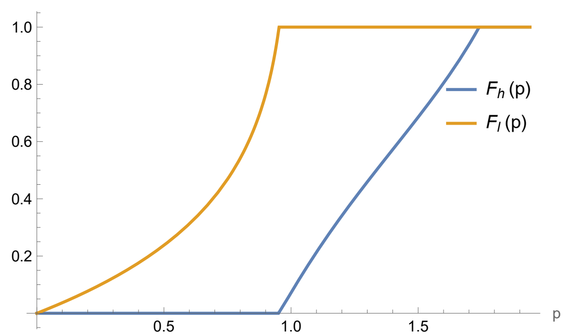


Figure 7: Bidding distributions for c=0.01

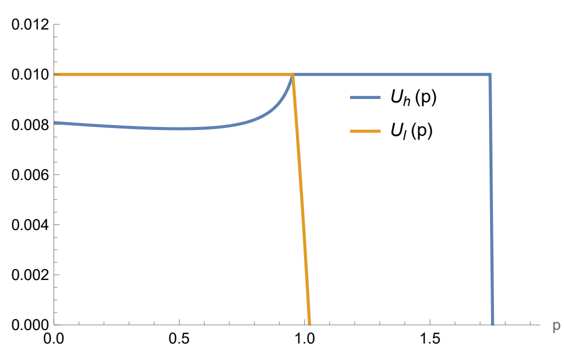


Figure 8: Value of bidding $p$ for c=0.01