

Entrepreneurship, Financiership, and Selection*

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Abstract

We develop an equilibrium model of the market for entrepreneurial finance, in which all agents have some personal wealth and a project whose quality is their private information. All agents choose whether to invest either as entrepreneurs or financiers, or to invest in storage technology. We find that a binding economy-level wealth constraint, which renders credit scarce, can create advantageous selection, where productive agents become entrepreneurs and unproductive agents become their financiers. If funding is easier to obtain, entrepreneurship also attracts unproductive agents. In our model, individual wealth and entrepreneurship are positively (negatively) correlated if financial market participation is complete (incomplete).

Keywords: Asymmetric information; credit constraints; entrepreneurial finance; financial market efficiency; start-up creation

JEL classification: D53; D82; G14; G30; L26

I. Introduction

It is widely recognized that innovative start-ups are the engines of job creation and economic growth, but that the market for entrepreneurial finance is fraught with market failure. These observations have led to extensive public intervention, aimed at promoting entrepreneurship and its finance. However, as evidence surveyed by Lerner (2009) suggests, policies that spur entrepreneurship are often unsuccessful for reasons that are not fully understood. To shed light on the challenges faced by policy-makers and the roles of different market imperfections, we construct a simple equilibrium

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model of entrepreneurial finance, which incorporates the major perceived reasons for market failures: asymmetric information, and individual and aggregate capital constraints.

A key message emerging from our paper is that many public interventions will simultaneously affect the outside options of entrepreneurs and financiers. Thus, they might have unintended consequences. For example, an injection of capital into a market, where some productive projects cannot be executed because of a lack of credit, will simultaneously make entrepreneurship more attractive and its finance less attractive. This can lead to adverse selection, where agents with low-quality projects prefer entrepreneurship rather than financing the ventures of others.

We also identify an aggregate wealth constraint as a crucial determinant of market efficiency. When the economy's total wealth is sufficient to implement all projects with positive net present value (NPV), interest rates tend to be sufficiently low to stimulate unproductive entrepreneurship. In contrast, a binding economy-level wealth constraint can induce advantageous selection. When aggregate credit is restricted, higher interest rates simultaneously discourage agents with low-quality projects from becoming entrepreneurs, and encourage them to invest in the projects of others.

Another implication of our model concerns the relationship between the wealth of agents and their entrepreneurship. The empirical body of literature has documented a positive relationship between individual wealth and entrepreneurship (e.g., Evans and Jovanovic, 1989; Black *et al.*, 1996; Gentry and Hubbard, 2004). However, basic adverse selection theory predicts the reverse (e.g., de Meza and Webb, 1987). Our model also shows that individual wealth and entrepreneurship might be negatively correlated, but only if there is sufficient wealth in the economy to induce some agents to opt out of the financial markets. If financial market participation is complete, there is a positive relationship between individual wealth and entrepreneurship.¹ However, in our model, there can be too much entrepreneurship, even if individual wealth and entrepreneurship are positively correlated.

We build on the well-established body of literature concerning entrepreneurial finance, with asymmetric information emerging from Stiglitz and Weiss (1981) and de Meza and Webb (1987). De Meza and Webb (1987, 1990, 1999) argue that the existence of credit constraints or asymmetric information, *per se*, does not constitute a solid rationale for subsidizing entrepreneurship or its finance.² Here, the same conclusion emerges

¹ In other words, our model predicts that it might be difficult to establish an unambiguous correlation between individual wealth and entrepreneurship. This is in line with Hurst and Lusardi (2004), who show that it is unlikely that wealth and entrepreneurship are positively related across all wealth classes.

² For an elegant generalization of this argument, see Boadway and Keen (2006).

within the equilibrium when the individual wealth levels of agents are moderate, and the economy-level wealth constraint is not binding. When it does bind, however, the increases in individual wealth promote productive entrepreneurship, and there could be a case for subsidizing business creation. Like us, de Meza and Webb (1999) show that excessive entrepreneurial activity can coexist with a positive relationship between individual wealth and entrepreneurship. However, in this paper, the coexistence arises without moral-hazard considerations.

Our paper also ties in with the body of literature that concerns occupational choice in the presence of frictions in the financial market. In particular, the contemporaneous works by Inci (2006) and Ghatak *et al.* (2007) consider general equilibrium models where privately informed agents, who face credit constraints, choose between entrepreneurship and paid employment.³ We focus on both the decision to participate in the financial market and the choice between becoming an entrepreneur or a financier. Our relatively sparse model yields a rich set of outcomes and policy implications. Even though it ignores the labor market, the model generates an outside-option mechanism, which is similar to that in Inci (2006) and Ghatak *et al.* (2007). This mechanism drives selection into entrepreneurship and, in this paper, into “financiership”. Moreover, we identify an economy-wide wealth constraint, which determines whether selection creates adverse or advantageous effects.

To the best of our knowledge, Boyd and Prescott (1986) and Shleifer and Wolfenzon (2002) are the only previous authors to have considered a genuine choice between investing as an entrepreneur or a financier. Whereas the work of Shleifer and Wolfenzon (2002) has little to do with our analysis, the paper of Boyd and Prescott (1986) is closely related to our study. However, our model is simpler in that we do not allow for information-producing financial intermediaries. Unlike us, Boyd and Prescott (1986) do not allow for a binding aggregate wealth constraint. We show that when an aggregate wealth constraint binds, or when agents are sufficiently wealthy, there is often no need for information provision by financial institutions, because the markets are efficient. We also highlight the comparative statics over the wealth of agents.

Our study is inspired by the above-mentioned papers and also by Holmström and Tirole (1997, 1998), Caballero and Krishnamurthy (2001), and Aghion *et al.* (2004). These authors emphasize that both microlevel and

³ Yet another related contemporaneous study is Antunes *et al.* (2008), where agents differing in their managerial talent (which is common knowledge) not only choose between paid employment and entrepreneurship but also between investing their initial wealth in their own firm or in other firms (via financial intermediaries). Antunes *et al.* focus on explaining cross-country variations in economic performance by using exogenous intermediation costs and investor protection.

economy-level financial constraints influence the behavior of the financial market. From this perspective, our study is also linked to the body of literature concerning the effects of financial liberalization in the presence of adverse selection (e.g., Giannetti, 2007; Sengupta, 2007).

In Section II, we present the model. In Section III, we show how to construct equilibria, and we analyze their existence and efficiency, relegating most of the algebra to the Appendices. To clarify the contribution of our paper, and to provide some intuition for our results, in Section IV, we compare our model to the standard partial equilibrium model with an unlimited supply of finance. We analyze the relationship between wealth and entrepreneurship in Section V. Section VI is devoted to the policy implications, and we conclude in Section VII.

II. The Model

The economy is populated by a $[0,1]$ continuum of risk-neutral agents, each with access to a project of size I and with personal wealth (cash) of $0 < A < I$. The project returns have a two-point distribution. A proportion h ($0 < h < 1$) of agents are high (H) types, each endowed with a positive NPV project, and the rest are low (L) types with a negative NPV project. As, for example, in Holmström and Tirole (1997), we assume that $p_H R_H > I > p_L R_L$ and $R_L > R_H$, where p_t is the success probability and R_t is the return (conditional on success) of an entrepreneur of type t , $t \in \{H, L\}$. Failed projects yield zero. In other words, the project return distributions are characterized by second-order stochastic dominance (but not mean-preserving spread).⁴ Following the convention in the literature (e.g., de Meza and Webb, 1987; Boadway and Keen, 2006), we assume that agents are protected by limited liability, that the personal wealth of agents is common knowledge, but that the project type is information private to them.

In our model, an agent can choose between the following options: (a) be an entrepreneur and invest her wealth in her own project;⁵ (b) be a financier and use her wealth to finance the projects of others; (c) invest her wealth in storage technology. That is, each agent chooses action $a \in \{e, f, s\}$ from the agents' common action space, where e , f , and s denote becoming an

⁴ In Section 3.3 we briefly consider the first-order stochastic dominance and mean-preserving spread of project return distributions ($R_H = R_L$ and $p_H R_H = p_L R_L$, respectively).

⁵ Agents that are entrepreneurs must invest their wealth entirely in their own projects, in equilibrium. As in de Meza and Webb (1987), for example, it is cheaper for H-type entrepreneurs to use their own rather than others' funds. Consequently, L-type entrepreneurs have no other option than to follow and invest all their wealth in their own projects. Note that the agents have no illiquid outside wealth that could be pledged as collateral to facilitate the emergence of a separating equilibrium (such as, for example, in Bester, 1987).

entrepreneur, becoming a financier, and investing in the storage technology, respectively.

As entrepreneurs need to raise external funds from other potential entrepreneurs, the aggregate initial wealth of agents restricts investment possibilities. We consider an economy to be wealth-constrained if the total wealth of agents $A_{\text{agg}} \equiv A \int_0^1 di$ is insufficient to finance all H-type projects $hI_{\text{agg}} \equiv hI \int_0^1 di$ (i.e., $A_{\text{agg}}/I_{\text{agg}} = A/I < h$). Correspondingly, an economy is non-wealth-constrained if the total wealth exceeds the financing needs of all H-type projects (i.e., $A_{\text{agg}}/I_{\text{agg}} = A/I \geq h$).

There are no financial institutions that process information. So, the financial market in our model could be interpreted as a frictionless (credit) market, a passive mutual fund, or a microfinance institution.⁶ The market collapses to autarky when all agents resort to the storage technology, and there are no entrepreneurs or financiers. In an efficient equilibrium, all H-type projects – or as many of them as possible – are financed, whereas no L-type projects receive finance. Correspondingly, in an inefficient equilibrium, at least some L-type projects are carried out.

We focus on risky debt contracts, which give a financier a fixed repayment in the case of success, and zero otherwise.⁷ Following some other common practices in the literature, we assume that storage technology is perfect, with a zero rate of return.⁸ We also assume that markets must clear in equilibrium, and that agents cannot publicly destroy their individual wealth before investing.

The timing of events is summarized as follows. First, each agent decides whether to invest her individual wealth (A) in her own project, in the projects of others, or in storage. If she initiates her own project, the rest of the required funds ($I - A$) must be raised from the agents who became

⁶ A more guarded interpretation is a group within a microfinance institution; however, even the formation of such groups can be thought of as being frictionless (see Eeckhout and Munshi, 2010).

⁷ The focus on debt financing in the literature is often motivated by its prevalence in practice. We share this motivation. For example, according to the European Commission (2009) and Robb and Robinson (2009), debt constitutes the main form of external finance for start-ups. However, note that risky debt contracts can also be optimal in plausible circumstances (e.g., when only payments from entrepreneurs to financiers are verifiable and entrepreneurs cannot hide income in case of default; see, for example, de Meza and Webb, 1999) or when project success is verifiable but project returns are not (or when returns are only partially verifiable, such as, for example, in Bolton and Scharfstein, 1990) and a contract cannot specify a positive reward for refraining from investing. We leave for future research the study of the consequences of richer verifiability assumptions for optimal security design in our set-up, where agents can choose their financial occupation and the economy-level wealth constraint can bind. However, at least to the extent that the equilibria we investigate are efficient, there is no room for more efficient contracting, even under richer verifiability assumptions.

⁸ In a longer working paper version of this paper (Takalo and Toivanen, 2006), we allow for imperfect storage technology.

financiers. Debt-contract terms stipulate the payment from entrepreneur to investors in the case of success. Second, entrepreneurs execute their projects. Project returns are realized, and successful entrepreneurs compensate financiers according to the debt contract.

III. Equilibria

We look for Bayesian equilibria, where the pure strategies of agents are functions from their common type space $\{H, L\}$ into their common action space $\{e, f, s\}$, and agents correctly anticipate such type-contingent strategies of the other agents. It is possible that at least some agents randomize over their pure strategies in equilibrium. Because there is a continuum of agents, we model mixed strategies using a distributional approach, where a proportion μ_t of t -type agents use the pure strategy of becoming an entrepreneur, a proportion χ_t use the pure strategy of investing in storage, and a proportion $1 - \mu_t - \chi_t$ use the pure strategy of becoming a financier for some $\mu_t, \chi_t \in [0, 1]$, $t \in \{H, L\}$. In equilibrium, the proportions μ_t and χ_t satisfy the indifference requirements between pay-offs from the pure strategies. The expected profit for an agent of type t from action a is denoted by $\pi(a, t)$.

The equilibria consist of four conditions. The first arises from the individual rationality (IR) constraints. All agents compare the expected profits from participating in the financial market, as either an entrepreneur or a financier, to investing in storage. Because the efficiency of storage technology is independent of the agent's type (i.e., $\pi(s, H) = \pi(s, L) = A$), the IR constraints can be written as

$$\pi(a, t) \geq A, \text{ for } a \in \{e, f\}, \text{ and } t \in \{H, L\}. \quad (1)$$

Second, the incentive compatibility (IC) constraints guide the agent's choice between investing as an entrepreneur or a financier:

$$\pi(a, t) \geq \pi(a', t), \text{ for } a, a' \in \{e, f\}, a \neq a', \text{ and } t \in \{H, L\}. \quad (2)$$

The third condition is that the supply of funds from financiers equals the demand for funds by entrepreneurs:

$$\begin{aligned} & (I_{\text{agg}} - A_{\text{agg}})[\mu_H h + \mu_L(1 - h)] \\ & = A_{\text{agg}}[(1 - \mu_H - \chi_H)h + (1 - \mu_L - \chi_L)(1 - h)] \end{aligned} \quad (3)$$

The left-hand side of equation (3) captures the demand. Each entrepreneur demands $I - A$ of funds, and the equilibrium mass of entrepreneurs is $[\mu_H h + \mu_L(1 - h)] \int_0^1 di$. Similarly, the supply of funds from financiers is given by the right-hand side of equation (3).

Table 1. *Potential equilibria*^a

	$\mu_L = 0$	$0 < \mu_L < 1$	$\mu_L = 1$
$\mu_H = 0$	Autarky	Not possible	Not possible
$0 < \mu_H < 1$	$H^{ef}L^f$	$H^{ef}L^{ef}$	$H^{ef}L^e$
$\mu_H = 1$	H^eL^{fs}	H^eL^{ef}, H^eL^{efs}	Not possible

^aThe proportion of *t*-type agents who become entrepreneurs in equilibrium is denoted by μ_t . The superscripts “e”, “f”, and “s” denote entrepreneurs, financiers, and users of storage, respectively.

Finally, the (expected) payments by successful entrepreneurs must equal the (expected) payments received by financiers. That is, it must hold that

$$R_B [\mu_H h p_H + \mu_L (1 - h) p_L] = R_F [(1 - \mu_H - \chi_H) h + (1 - \mu_L - \chi_L) (1 - h)]. \tag{4}$$

In equation (4), R_B is the fixed payment that an entrepreneur has promised to pay back in case of success, and R_F is the expected payment received by a financier, which, because of the law of large numbers, equals the realized payment. The term in square brackets on the right-hand side of equation (4) is the equilibrium proportion of financiers; the term on the left-hand side gives the equilibrium proportion of successful entrepreneurs.

In equilibrium, the types and actions of other agents affect an agent’s pay-offs only through the cost of borrowing as an entrepreneur (R_B) and the repayment received as a financier (R_F). Hence, expected profits from entrepreneurship can be written as

$$\pi(e, t) = p_t (R_t - R_B) \quad \text{for } t \in \{H, L\}. \tag{5}$$

The expected profits from investing as a financier can be written as

$$\pi(f, t) = R_F \quad \text{for } t \in \{H, L\}. \tag{6}$$

Table 1 gives a 3×3 matrix of potential equilibria.⁹ It is immediately clear that three of the nine cannot exist. If no H-type agent becomes an entrepreneur, the IR constraints of the potential financiers are violated. Similarly, because $A_{agg} < I_{agg}$, it is impossible that all agents will become entrepreneurs. Besides autarky, the remaining six potential equilibria consist of five cases in which financial markets emerge as an equilibrium outcome. We name these five cases according to the actions the agents choose. For example, in equilibrium H^eL^{fs} , all H-type agents become entrepreneurs, and L-type agents are divided into financiers and storage users.

Both efficient equilibria are in the first column of Table 1. The other three equilibria, with financial markets, are inefficient, because at least

⁹ These nine categories can be divided further according to whether agents participate or not.

some L-type agents become entrepreneurs. It is straightforward but tedious to solve the range of parameters where equations (1)–(4) hold for all five equilibria with financial markets. Hence, we construct only the equilibrium $H^e L^{ef}$ in detail in the main text. The remaining equilibria are described graphically, with detailed calculations in the Appendices.

Example: $H^e L^{ef}$

In $H^e L^{ef}$, $\mu_H = 1$, $\mu_L \in (0, 1)$, and $\chi_H = \chi_L = 0$. That is, all H-type agents are entrepreneurs, L-type agents become either entrepreneurs or financiers, and no one invests in the storage technology. This outcome corresponds to the decentralized market equilibrium in Boyd and Prescott (1986). It is also similar to a pooling equilibrium, which is familiar from the standard partial equilibrium model, in the sense that there is no aggregate shortage of funds and L-type agents are inefficiently attracted to entrepreneurship.

Because financial market participation is complete in this equilibrium, we require that the IR constraints of agents are satisfied. That is, from equations (1) and (6), we obtain

$$R_F \geq A. \quad (7)$$

L-type agents separate into the two occupations, so that their IC constraint must hold with equality. From equations (2), (5), and (6), this means that

$$p_L(R_L - R_B) = R_F. \quad (8)$$

The left-hand side of equation (8) gives the expected return of an L-type agent from becoming an entrepreneur, and the right-hand side gives the expected return from becoming a financier.

Because all H-type agents prefer being entrepreneurs to being financiers, their expected return from entrepreneurship must be at least as large as that of becoming a financier. That is, from equations (2), (5), and (6), the following must hold:

$$p_H(R_H - R_B) \geq R_F. \quad (9)$$

When $\mu_H = 1$ and $\chi_H = \chi_L = 0$ are inserted into equations (3) and (4), the aggregate demand and supply for entrepreneurial finance is balanced when

$$(J_{agg} - A_{agg})[h + \mu_L(1 - h)] = A_{agg}(1 - \mu_L)(1 - h), \quad (10)$$

and the (expected) repayments from successful entrepreneurs equal the payments received by their financiers when

$$R_B [hp_H + \mu_L(1 - h)p_L] = R_F(1 - \mu_L)(1 - h). \quad (11)$$

Conditions (8), (10), and (11) determine the endogenous variables μ_L , R_B , and R_F . Solving first for the proportion of L-type entrepreneurs, μ_L from condition (10) gives

$$\mu_L^* = \frac{A_{\text{agg}} - hI_{\text{agg}}}{(1 - h)I_{\text{agg}}}.$$

Noting that $A_{\text{agg}} = A \int_0^1 di$ and $I_{\text{agg}} = I \int_0^1 di$, this can be rewritten as

$$\mu_L^* = \frac{A - hI}{(1 - h)I}. \tag{12}$$

Using equations (8), (11), and (12), we can solve for the equilibrium payments R_B^* and R_F^* :

$$R_B^* = \frac{(I - A)}{I [hp_H + (1 - h)p_L]} p_L R_L \tag{13}$$

and

$$R_F^* = \left\{ 1 - \frac{p_L(I - A)}{I [hp_H + (1 - h)p_L]} \right\} p_L R_L. \tag{14}$$

To seek the range of parameter values for which the IC constraint of H-type agents is satisfied, we substitute equations (13) and (14) for equation (9), obtaining

$$A \geq I - \frac{(p_H R_H - p_L R_L) I [hp_H + (1 - h)p_L]}{(p_H - p_L) p_L R_L}. \tag{15}$$

When equation (15) holds, the IR constraint of H-type agents is redundant, and we only need to ensure that the IR constraint of L-type agents holds. Inserting equation (14) into equation (7) yields

$$A \leq \frac{p_L R_L I h (p_H - p_L)}{I h (p_H - p_L) + p_L (I - p_L R_L)}. \tag{16}$$

When equation (16) holds, no L-type agent stores individual wealth.

The set of parameter values for which $H^e L^{\text{ef}}$ exists is shown in Figure 1, in the (A, h) -space, where the horizontal axis represents the individual wealth of agents ($A < I$) and the vertical axis represents the proportion of H-type agents ($h \leq 1$). The $h = A/I$ -diagonal divides the economy into wealth-constrained (above) and non-wealth-constrained (below). By definition, $H^e L^{\text{ef}}$ can only exist in a non-wealth-constrained economy where aggregate wealth is sufficient to finance the projects of all H-type agents. This can also be seen from equation (12); $\mu_L^* \geq 0$ implies that $A/I \geq h$. The H-type IC constraint (15) is a downward-sloping line in (A, h) -space, so that a relatively poor economy with a relatively small proportion of good projects fails to satisfy this constraint. The L-type IR

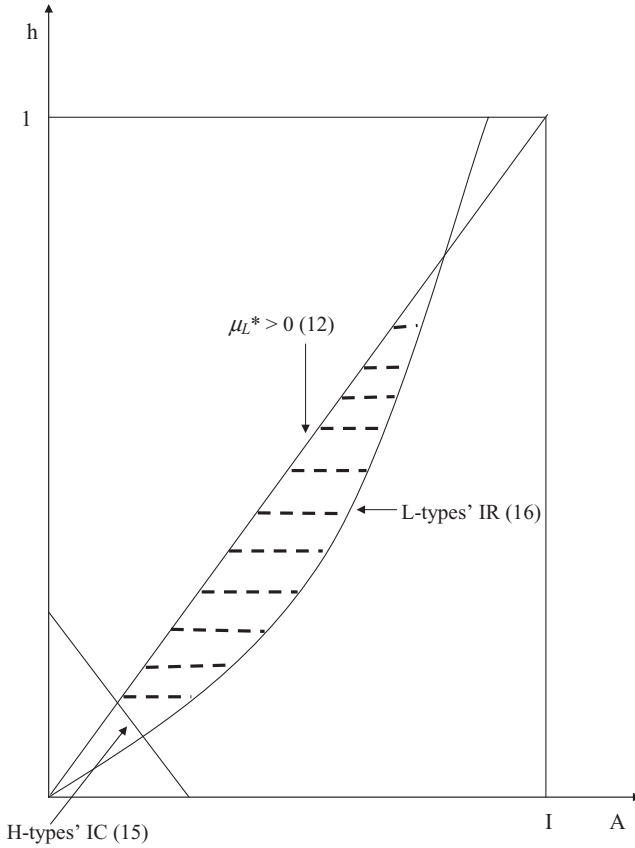


Fig. 1. Equilibrium $H^e L^{ef}$

constraint (16) is a monotonically increasing curve, which starts at the origin and cuts the diagonal once. Below the curve, some L-type agents prefer not to participate.

Let us consider how a small decrease in aggregate wealth affects the equilibrium outcome. Clearly, it reduces the funds available to entrepreneurs. Because in this equilibrium H-type agents prefer being entrepreneurs to being financiers and L-types are indifferent, a marginal change in wealth has no impact on the actions of H-type agents, but it must induce some L-type entrepreneurs to become financiers (μ_L^* declines). That is, the ratio of entrepreneurs to financiers diminishes, which drives down the compensation per financier (R_F^*). Because each entrepreneur needs to borrow more, R_B^* increases just enough to keep L-type agents indifferent.

This example illustrates how a decrease in aggregate wealth tightens the financial market, which can be beneficial in terms of efficiency. As in de Meza and Webb (1987), there is too much entrepreneurship, with marginal entrepreneurs being of low quality. These are driven out by higher lending rates. However, here, the problem of overinvestment emerges as part of the equilibrium. The total wealth of the economy relative to the proportion of high-quality projects is too large. Moreover, there is a positive relationship between the wealth and entrepreneurship of an individual agent. This is at odds with the prediction of de Meza and Webb (1987), but it has empirical appeal.

Existence and Efficiency of Equilibria

Following the procedure outlined in the previous subsection, we analyze in the Appendices the existence of the remaining equilibria, where financial markets endogenously emerge. Here, we present the results graphically, and we describe their efficiency properties. At the end of this subsection, we summarize our main results.

In Figure 2, we indicate the areas in which each equilibrium exists in the (A, h) -space. There are two key lines: the $h = A/I$ -diagonal and the vertical $\hat{A} \equiv p_L p_H (R_L - R_H) / (p_H - p_L)$ line. These are shown at full length, with the sections confining equilibria in bold. The other bold lines bounding equilibria come from the various IC and IR constraints of the agents. For example, in the middle of a non-wealth-constrained region, we have a part of the L-type IR constraint (16) familiar from the case $H^e L^{ef}$ of the previous subsection. Similarly, Figure 2 shows the part of the H-type IC constraint (15) that acts as an equilibrium boundary.

As Figure 2 shows, the diagonal not only determines whether the economy is wealth-constrained or not, but also borders many of the equilibria. To the left of the vertical \hat{A} -line, limited liability renders the pledgeable income of L-type agents higher than that of H-type agents.¹⁰ To the right of the \hat{A} -line, where the pledgeable income of H-type agents is higher than that of L-type agents, the equilibria are unique.

Let us first examine a non-wealth-constrained economy (below the diagonal in Figure 2). In the right-hand part, we find an efficient equilibrium, $H^e L^{fs}$. All H-type projects are financed, and L-type agents are indifferent between funding the H-types and investing in the storage technology. If $A \geq p_L R_L$, L-type agents trivially prefer investing in storage technology to entrepreneurship. Because of costly financing ($R_B^* > 0$), L-types

¹⁰ Pledgeable income, as defined by Holmström and Tirole (1997), is the maximum amount an entrepreneur can credibly promise to pay back to a financier.

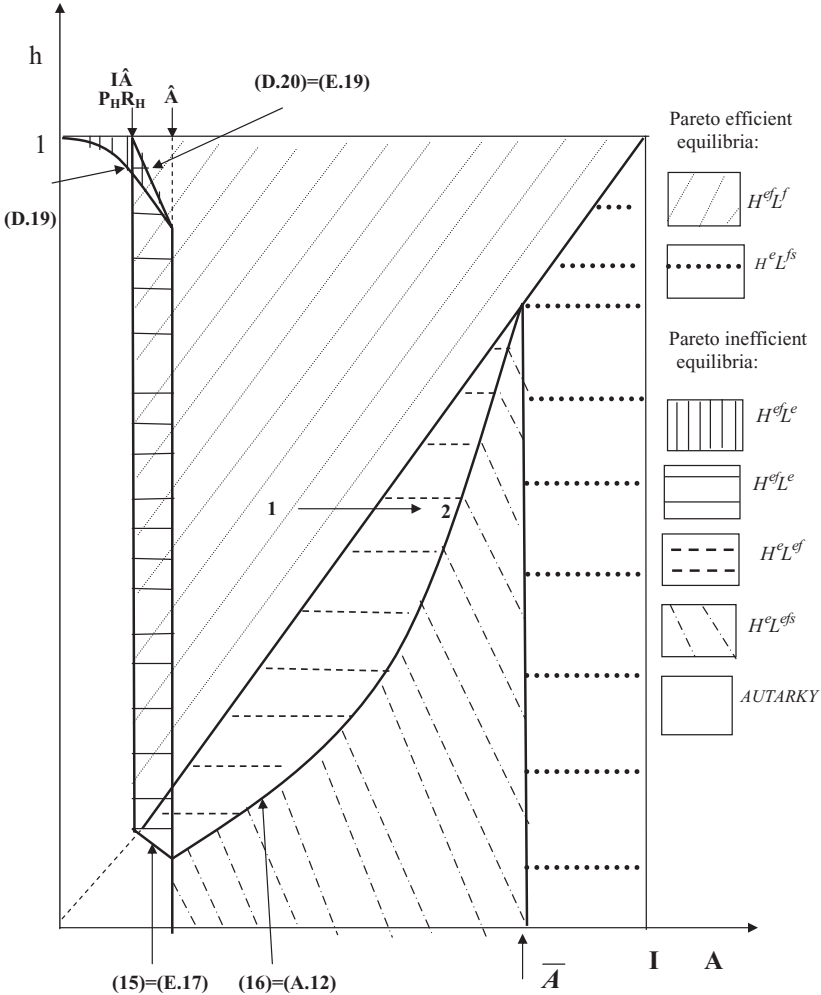


Fig. 2. Equilibria.

continue to find storage superior, even if their individual wealth is slightly less than the expected return on their project.

Once entrepreneurs have a sufficiently low stake in their projects (i.e., A is below the vertical \bar{A} -line, where $\bar{A} \equiv p_L(p_H R_L - I)/(p_H - p_L) < p_L R_L$), L-type agents are no longer discouraged from entrepreneurship. This results in an inefficient semi-separating equilibrium, where some L-type agents pool with H-type agents as entrepreneurs. Below the L-type IR curve (16), we have the equilibrium H^eL^{efs} , where some L-type agents opt for storage. Above the L-type IR curve, we find H^eL^{ef} of the previous

subsection. Nobody uses storage, even though available assets exceed the funding needs of H-type entrepreneurs, so that demand and supply of funds are equated, as some L-types become entrepreneurs. Autarky prevails to the left of the H-type IC constraint (15).

In a wealth-constrained economy (above the diagonal in Figure 2), the outcomes are quite different. In the middle and right parts, characterized by wealthier agents with a higher fraction of good projects, all L-type agents are financiers and H-type agents include both entrepreneurs and financiers. This $H^{ef}L^f$ equilibrium is efficient; the economy's total endowment is directed into positive NPV projects.

Entrepreneurship becomes relevant to L-type agents only when we reach the vertical \hat{A} -line. Between this and another vertical line, $\hat{A}/p_H R_H$, we have one to three equilibria. One is the same $H^{ef}L^f$ as to the right of the \hat{A} -line. There are also two inefficient equilibria: $H^{ef}L^{ef}$, where both agent types can be found among entrepreneurs and financiers, and $H^{ef}L^e$, where all L-type agents are entrepreneurs and H-type agents include both entrepreneurs and financiers. Interestingly, only $H^{ef}L^e$ survives to the left of the $\hat{A}(I/p_H R_H)$ -line. The equilibrium can be supported in an area where the proportion of H-type agents is high. With a lower proportion of H-type agents, the financial markets cease to operate because of adverse selection.

We summarize the above results in the following proposition.

Proposition 1. (a) *The equilibrium is autarky if the level of initial wealth is sufficiently low, and it is efficient if the level of initial wealth is sufficiently high.* (b) *In the intermediate range of initial wealth ($A \in [\hat{A}, \bar{A}]$), the equilibrium is efficient in a wealth-constrained economy and it is inefficient in a non-wealth-constrained economy.* (c) *The threshold level of wealth that prevents the market from collapsing and the threshold level of wealth that yields an efficient equilibrium are both higher in a non-wealth-constrained economy than in a wealth-constrained economy.* (d) *Multiple equilibria can only exist between $\hat{A}(I/p_H R_H)$ and \hat{A} .*

Proposition 1 shows that not only the individual wealth constraint but also the aggregate one matters. Most clearly, this can be seen from parts (b) and (c) of Proposition 1. In a wealth-constrained economy, $A \geq \hat{A}$ is a sufficient condition for an efficient equilibrium whereas in a non-wealth-constrained economy, it is only a sufficient condition to avoid a collapse of the market to autarky.

Proposition 1 also suggests that changes in the wealth of agents might also change the type of equilibrium. A decrease in wealth might shift the economy from an efficient equilibrium to an inefficient equilibrium (e.g., from $H^e L^{fs}$ to $H^e L^{efs}$), or even to autarky (e.g., from $H^{ef}L^f$ to autarky).

Interestingly, in our model, an increase in wealth can also reduce efficiency. Increasing wealth might move an economy from an efficient $H^{ef}L^f$ equilibrium to an inefficient H^eL^{ef} (from point 1 to point 2 in Figure 2).¹¹

Another implication of Proposition 1 is that there is no need for financial intermediaries to produce information in the efficient equilibria. However, Boyd and Prescott (1986) show that efficiency-improving financial intermediation could endogenously arise in the range of parameter values where equilibrium H^eL^{ef} prevails. Clearly, in all other inefficient equilibria, it is also possible for intermediaries to mitigate informational problems. However, the question of whether they can endogenously emerge beyond H^eL^{ef} is left for future research.

There can be, at most, three equilibria in the region identified by part (d) of Proposition 1. Recall that to the left of the \hat{A} -line, the pledgeable income of L-type agents is higher than that of H-type agents, so that even the high costs of borrowing cannot discourage L-type agents from entrepreneurship. This helps to explain the existence of an inefficient equilibrium $H^{ef}L^e$, where the cost of borrowing is relatively high and the return on lending is relatively low. Even though all L-type agents are entrepreneurs, this equilibrium can survive if the proportion of L-type agents is low enough. There is also an efficient equilibrium $H^{ef}L^f$, where lending is sufficiently profitable to induce all L-type agents to become financiers. However, as the return on lending is the lower, the larger the share of a project that is funded externally, $H^{ef}L^f$ cannot be supported to the left of the $\hat{A}(I/p_H R_H)$ -line. Because both $H^{ef}L^e$ and $H^{ef}L^f$ are pure strategy equilibria with respect to the actions of L-type agents, there must also exist a corresponding mixed strategy equilibrium $H^{ef}L^{ef}$.

IV. Comparison to the Case with an Unlimited Supply of Financial Capital

Let us briefly compare our results to those from the standard partial equilibrium models, where aggregate wealth is not an issue.¹² In these models, there is typically free entry of financiers with unlimited access to financial

¹¹ None the less, such an increase in wealth can increase social welfare. Up to the diagonal $h = AI$, wealth increases improve welfare by increasing H-type entrepreneurship. However, to the extent that the increase continues past the diagonal, it reduces efficiency. As a result, whether the net welfare effect of the individual wealth increase is positive or negative depends on the relevant parameter values and the distance of points 1 and 2 from the diagonal. Roughly speaking, if the starting point (point 1) is sufficiently close to the diagonal and the ending point (point 2) extends well beyond the diagonal, the increase in initial wealth will reduce welfare as well as efficiency.

¹² A more detailed comparison can be found in the working paper version of this paper (Takalo and Toivanen, 2006).

capital (but without a project of their own), which makes the supply of funding perfectly elastic.

In their leading paper, Stiglitz and Weiss (1981) consider the mean-preserving spread of project return distributions. In our notation, this means that $p_H R_H = p_L R_L \equiv \bar{R}$, and thus that the pledgeable income of L-type entrepreneurs exceeds that of H-type entrepreneurs for all relevant parameter values (for $A < \hat{A} = \bar{R}$). Hence, it is not surprising that we obtain Stiglitz–Weiss-type results in the region where the aggregate wealth constraint does not bind and $A < \hat{A}$. Financial markets collapse when the proportion of H-type entrepreneurs is low enough. With a higher proportion of H-types, the average quality of entrepreneurs is sufficient to sustain a pooling equilibrium, as in Stiglitz and Weiss (1981).

In another influential article, de Meza and Webb (1987) assume the first-order stochastic dominance of project return distributions, which, in our model, is equivalent to assuming that $p_H > p_L$ but that $R_H = R_L$. This would render the pledgeable income of H-type entrepreneurs larger than that of L-type entrepreneurs for any A (i.e., for $A \geq \hat{A} = 0$). Again, we obtain de Meza–Webb-type overinvestment results in a non-wealth-constrained economy when $A \in [\hat{A}, \bar{A}]$. There is too much entrepreneurship, as at least some L-types pool with H-types and become entrepreneurs. If the entrepreneurs have a high stake in their projects ($A > \bar{A}$), an efficient separating equilibrium emerges where all L-types invest in storage.¹³

In summary, the partial equilibrium models suggest that the financial markets are inefficient because competitive financiers with unlimited investment funds drive interest rates too low, which encourages unproductive entrepreneurship. Similarly, when the aggregate wealth constraint does not bind in our model, lending and borrowing rates are relatively low, which not only encourages entrepreneurship but also discourages its finance. In contrast, when the aggregate wealth constraint binds, the relative scarcity of funds raises the interest rates, which can be conducive to financial market efficiency, particularly in the intermediate individual wealth range ($A \in [\hat{A}, \bar{A}]$). When $A \geq \hat{A}$, the pledgeable income of H-type entrepreneurs exceeds that of L-type entrepreneurs. In such a case, the higher opportunity cost of entrepreneurship discourages mainly L-type entrepreneurs, which improves the quality of the entrepreneurial pool. The same logic need not apply when $A < \hat{A}$, because then the pledgeable income of L-type entrepreneurs is higher than that of H-type entrepreneurs.

¹³ This is reminiscent of de Meza and Webb (1990), where a separating equilibrium emerges when entrepreneurs are sufficiently risk-averse.

The higher opportunity cost first affects the choices of H-types, causing an adverse effect on the average quality of entrepreneurs.

V. Wealth and Entrepreneurship

The aggregate wealth constraint also affects the relationship between the wealth and entrepreneurship of an individual agent. In a non-wealth-constrained economy, increases in individual wealth might raise the economy out of autarky and cause the efficient exit of L-type entrepreneurs in equilibrium $H^e L^{efs}$. However, they also stimulate the inefficient entry of L-type entrepreneurs in equilibrium $H^e L^{ef}$ (see Section III). In a wealth-constrained economy, individual wealth is positively associated with the efficient entry of H-type entrepreneurs in all equilibria, except in $H^{ef} L^{ef}$, where H-type entrepreneurs are replaced by L-types as wealth rises. None the less, entrepreneurship in $H^{ef} L^{ef}$ is increasing in aggregate wealth.

Summarizing, we have a clear relation between the wealth and entrepreneurship of an agent.

Proposition 2. *(a) Individual wealth and entrepreneurship are (weakly) negatively correlated if storage is used. In this case, increases in individual wealth lead to efficient exit. (b) Individual wealth and entrepreneurship are (weakly) positively correlated if storage is not used. In this case, increases in individual wealth lead to inefficient entry in a non-wealth-constrained economy and to efficient entry in a wealth-constrained economy when $A \geq \hat{A}$.*

The negative relationship between individual wealth and entrepreneurship arises here for the same reason as in the partial equilibrium models. As the stake in one's own project rises, investing in storage rather than one's own project becomes more attractive for agents with low-quality projects. In our model, this can only occur if the level of aggregate wealth is so high that some agents prefer to opt out of the financial markets. If all the economy's assets are invested in entrepreneurial projects, the wealth and entrepreneurship of an individual agent are positively correlated. Moreover, increases in individual wealth can induce the entry of H-type entrepreneurs if the economy-level wealth constraint binds.

Note that we assume perfect storage technology. Clearly, the attractiveness of the storage technology as an investment option depends on its efficiency. When there is no storage technology, participation in the financial market must be complete, and individual wealth and entrepreneurship must be positively correlated (see Takalo and Toivanen, 2006).

VI. Policy Implications

Although there are several limitations¹⁴ to our simple model, we boldly offer some policy recommendations concerning the tricky subject of promoting entrepreneurship. Policy-makers often view access to finance as one of the key problems facing start-ups (see, for example, the European Commission, 2003, 2008, 2009; UK Government, 2008). Our findings refine the argument advanced by de Meza and Webb (1987, 1999) that the problem of access to finance need not be a reason to subsidize entrepreneurs or their financiers. In our model, there is too much lending and entrepreneurship in the intermediate individual wealth range ($A \in [\hat{A}, \bar{A}]$) when the aggregate wealth constraint of an economy is not binding. This applies even if business creation is increasing in the level of individual wealth. However, when we consider the same intermediate wealth range of a wealth-constrained economy, it turns out that productive entrepreneurs are held back by insufficient individual wealth. Moreover, insufficient individual wealth can lead to autarky in both wealth-constrained and non-wealth-constrained economies. Hence, a case for subsidies might arise.

To fix ideas, we could interpret the wealth-constrained region of our model as representing capital-constrained emerging economies with plenty of investment opportunities, and the non-wealth-constrained region as developed countries with cash but lacking in opportunities. In this case, our model suggests that the direct public funding of entrepreneurship is more likely to work in emerging economies than in the richer economies. The same applies to investment subsidies to financiers. However, note that we do not allow for the formation of financial intermediaries that provide information. While such sophisticated financiers should, as indicated by Boyd and Prescott (1986), improve efficiency, at least in the intermediate individual wealth range of developed economies, they could, on average, hamper rather than facilitate access to finance, because entrepreneurs with bad projects are denied finance.¹⁵

Subsidies are not the only policy tool for encouraging entrepreneurship. For example, the European Commission (2008) advances 10 principles for a European small- and medium-sized enterprise (SME) policy. The principles include goals such as “help SMEs to benefit more from the opportunities offered by the Single Market”, “promote the upgrading of skills in SMEs”, and “encourage and support SMEs to benefit from the

¹⁴ For instance, future work should consider more than two types of agents, heterogeneity in the wealth of agents, the formation of a coalition of financiers, a richer contracting space, and a more dynamic environment with capital accumulation via consumption and saving decisions.

¹⁵ Moreover, adverse selection can create excessive entrepreneurial entry, even in the presence of specialized start-up financiers (see Keuschnigg and Bo Nielsen, 2007).

growth of markets". Our analysis provides a more optimistic view of the prospects for such initiatives than for funding interventions in developed economies. In our model, higher entrepreneurial quality always improves welfare and efficiency, even if it leads to an aggregate wealth constraint. Keeping an agent's wealth constant, an increase in h either yields more successful projects within the initial equilibrium or results in a more efficient equilibrium. Similarly, our model suggests that increases in success probabilities or profits conditional on success are generally conducive to welfare, although they can lead to inefficiencies in certain circumstances.

VII. Conclusions

We study whether, despite asymmetric information and capital constraints, markets for entrepreneurial finance can endogenously emerge in equilibrium, and we examine the efficiency of the eventual markets. In our model, all agents have investment opportunities whose quality is their private information but encounter capital constraints. They can choose whether to invest their personal wealth in their own project, in the ventures of others, or in storage technology. We identify an economy-level wealth constraint as an important determinant of market efficiency. When the constraint is binding, it reduces the supply of credit and raises its cost. This creates advantageous selection, where agents with productive projects become entrepreneurs and those with unproductive projects become their financiers. In contrast, when credit is plentiful, low interest rates tend to result in overinvestment.

In our model, business creation and individual wealth can be positively correlated, but this need not provide a rationale for subsidizing entrepreneurs or their financiers. This result is similar in spirit to that of de Meza and Webb (1999), but we do not need to invoke moral-hazard considerations. We also find that it might be beneficial to subsidize business creation when the aggregate wealth constraint is binding.

Appendix A: H^eL^{ef} and H^eL^{efs}

In these appendices, we construct the remaining equilibria (H^eL^{ef} is characterized in the main text). For each equilibrium, we present the constraints, the equilibrium values of endogenous variables, and the equilibrium-existence conditions. We shorten the exposition by using the following notations: $\Delta p \equiv p_H - p_L$, $\Delta R \equiv R_L - R_H$, $\gamma \equiv p_H R_H - I$, $\lambda \equiv I - p_L R_L$, and $\Delta W \equiv \gamma + \lambda = p_H R_H - p_L R_L$. Because our approach to solving the model is rather mechanical, we provide a fair amount of detail in our solution for the first case, but somewhat less detail subsequently.

H^eL^{ef} is described in Section III, so we here characterize H^eL^{efs} and explain its relation to H^eL^{ef} . In H^eL^{efs} , all H-type agents are entrepreneurs and L-type agents are indifferent as to entrepreneurship, financing, and using the storage technology (i.e., $\mu_H = 1$, $\mu_L \in (0, 1)$, $\chi_H = 0$ and $\chi_L \in (0, 1)$). That is, the only difference compared to H^eL^{ef} is that χ_L is strictly positive. Hence, the IR constraints of agents (7) must hold as an equality, that is,

$$R_F = A. \tag{A1}$$

The IC constraints of agents are as in equations (8) and (9), that is,

$$p_L(R_L - R_B) = R_F \tag{A2}$$

and

$$p_H(R_H - R_B) \geq R_F. \tag{A3}$$

The economy-level “budget constraint” (10) becomes

$$(1 - \mu_L - \chi_L)(1 - h)A_{agg} = [h + \mu_L(1 - h)](I_{agg} - A_{agg}). \tag{A4}$$

Similarly, the equilibrium condition for repayment flows (11) is transformed to

$$[hp_H + \mu_L(1 - h)p_L]R_B = R_F(1 - \mu_L - \chi_L)(1 - h). \tag{A5}$$

A system consisting of equations (A1)–(A2) and (A4)–(A5) determines the values of the endogenous variables R_B , R_F , χ_L , and μ_L . Using equation (A1), $R_F^* = A$. Using this in equation (A2) and solving for R_B , we obtain

$$R_B^* = \frac{p_L R_L - A}{p_L}. \tag{A6}$$

Inserting equations (A1) and (A6) into equation (A5), and solving equations (A4) and (A5) for the remaining two endogenous variables, χ_L and μ_L , yields

$$\mu_L^* = \frac{h}{1 - h} \left[\frac{(p_L R_L - A)\Delta p}{p_L \lambda} - 1 \right] = \frac{h}{1 - h} \left(\frac{R_B^* \Delta p}{\lambda} - 1 \right) \tag{A7}$$

and

$$\chi_L^* = \frac{1}{1 - h} \left[1 - \frac{(p_L R_L - A)\Delta p I h}{p_L \lambda A} \right] = \frac{1}{1 - h} \left[1 - \frac{R_B^* \Delta p I h}{\lambda A} \right], \tag{A8}$$

where the last equalities come from equation (A6).

The equilibrium exists if χ_L^* and μ_L^* , as given by equations (A7) and (A8), satisfy our initial assumptions $\mu_L \in (0, 1)$ and $\chi_L \in (0, 1)$, and if the IC and IR constraints of agents are satisfied with R_B^* , as given by

equation (A6). The first four existence conditions are

$$\mu_L^* < 1 \Leftrightarrow A > p_L \left(R_L - \frac{\lambda}{h\Delta p} \right), \tag{A9}$$

$$\mu_L^* > 0 \Leftrightarrow A < p_L \left(R_L - \frac{\lambda}{\Delta p} \right) = \hat{A} + \frac{p_L \gamma}{\Delta p} = \bar{A}, \tag{A10}$$

$$\chi_L^* < 1 \Leftrightarrow A < \frac{p_L R_L I \Delta p}{I \Delta p + p_L \lambda} = \frac{p_L R_L I \Delta p}{(p_H I) - p_L^2 R_L}, \tag{A11}$$

and

$$\chi_L^* > 0 \Leftrightarrow A > \frac{p_L R_L I h \Delta p}{I h \Delta p + p_L \lambda} = \frac{p_L R_L I h \Delta p}{I (p_L + h \Delta p) - p_L^2 R_L}. \tag{A12}$$

Because the L-type IC and IR constraints bind by equations (A1) and (A2), the fifth existence condition comes from equation (A3), the H-type IC constraint. If it is satisfied, the H-type IR (A1) also trivially holds. Inserting equations (A1) and (A6) into equation (A3) shows that the H-type IC constraint holds if

$$A \geq \hat{A}. \tag{A13}$$

Equations (A9)–(A13) define the range of parameters for which $H^e L^{efs}$ exists. Because the critical values of A in equations (A11) and (A12) are strictly larger than the respective critical values in equations (A10) and (A9), the binding values are given by equations (A10) and (A12). These, in turn, cross each other at the diagonal $h = A/I$. This means that $H^e L^{efs}$ only exists in a non-wealth-constrained economy. In (A, h) -space, $H^e L^{efs}$ exists in the area between the vertical lines (A13) and (A10), and below the curve (A12) – which is identical to equation (16) – as depicted in Figure 2.

When equation (A12) is violated, the H-type IC constraint changes from equation (A13) to equation (15). Thus, $H^e L^{ef}$ exists in the range of parameters described in Section III, that is, in the area shaped by curve (A12), the downward-sloping line (15), and the $h = A/I$ -diagonal. The vertical \hat{A} -line, curve (A12) and equation (15) cross at the same point where $h = h_1 \equiv \hat{A} \lambda / I \Delta W$.

Appendix B: $H^{ef} L^f$ and $H^{efs} L^{fs}$

We first prove that $H^{efs} L^{fs}$ cannot exist. In this equilibrium, $\mu_L = 0$ and μ_H, χ_H and $\chi_L \in (0, 1)$. The equilibrium is constrained by the following five conditions.

$$R_F \geq A \tag{B1}$$

$$p_L(R_L - R_B) \leq R_F \tag{B2}$$

$$p_H(R_H - R_B) = R_F \tag{B3}$$

$$[(1 - \mu_H - \chi_H)h + (1 - \chi_L)(1 - h)] A_{agg} = \mu_H h(I_{agg} - A_{agg}) \tag{B4}$$

$$h\mu_H p_H R_B = R_F [(1 - \mu_H - \chi_H)h + (1 - \chi_L)(1 - h)] \tag{B5}$$

Condition (B1) arises from the IR constraints of both types of agent, and conditions (B2) and (B3) arise from the IC constraints of L-type and H-type agents, respectively. Conditions (B4) and (B5) balance the supply and demand for funds.

In $H^{efs}L^{fs}$, condition (B1) holds with equality. Solving condition (B4) for μ_H yields

$$\mu_H = \frac{A}{hI} [1 - \chi_H h - \chi_L(1 - h)]. \tag{B6}$$

Using equations (B3) and (B6) in equation (B5), we can write R_B as

$$R_B^* = \frac{R_H(I - A)}{I}. \tag{B7}$$

Inserting equation (B7) into equation (B3) gives

$$R_F^* = p_H R_H \frac{A}{I}. \tag{B8}$$

Because R_F^* in equation (B8) is strictly larger than A , the initial assumption that condition (B1) binds is invalid. This means that $H^{efs}L^{fs}$ cannot exist.

However, $H^{ef}L^f$ allows for the inequality in condition (B1). This equilibrium can be characterized by setting $\chi_H = \chi_L = 0$ in equation (B6). As a result,

$$\mu_H^* = \frac{A}{hI}. \tag{B9}$$

Equation (B9) gives the following two equilibrium-existence conditions.

$$\mu_H^* < 1 \Leftrightarrow A < hI \tag{B10}$$

$$\mu_H^* > 0 \Leftrightarrow A > 0 \tag{B11}$$

Using equations (B7) and (B8), the L-type IC constraint (B2) can be written as

$$A \geq \frac{I}{p_H R_H} \hat{A}. \tag{B12}$$

Equations (B10) and (B12) define the range of parameters for which $H^{ef}L^f$ exists. As shown in Figure 2, the equilibrium exists in a wealth-constrained economy for $A \in [\hat{A}I/p_H R_H, I)$.

Appendix C: H^eL^{fs}

In this equilibrium, $\mu_H = 1$, $\mu_L = 0$, $\chi_H = 0$, and $\chi_L \in (0, 1)$. The five basic conditions constraining the equilibrium are

$$R_F = A, \quad (\text{C1})$$

$$p_L(R_L - R_B) \leq R_F, \quad (\text{C2})$$

$$p_H(R_H - R_B) \geq R_F, \quad (\text{C3})$$

$$(1 - \chi_L)(1 - h)A_{\text{agg}} = h(I_{\text{agg}} - A_{\text{agg}}), \quad (\text{C4})$$

and

$$hp_H R_B = R_F(1 - \chi_L)(1 - h). \quad (\text{C5})$$

Conditions (C1) and (C2) arise from the IR and IC constraints of L-type agents, respectively, and condition (C3) arises from the IC constraint of H-type agents. Together with condition (C1), condition (C3) also implies that the IR constraint of H-type agents is satisfied. Conditions (C4) and (C5) balance the supply and demand for funds.

By substituting the equilibrium value of R_F from condition (C1) into condition (C5), the other endogenous variables, χ_L and R_B , can be solved from conditions (C4) and (C5). These are given by

$$\chi_L^* = \frac{A - hI}{A(1 - h)} \quad (\text{C6})$$

and

$$R_B^* = \frac{I - A}{p_H}. \quad (\text{C7})$$

From equation (C6), we see that $\chi_L^* < 1$ with the assumption that $A < I$. Similarly, inserting equations (C1) and (C7) into equation (C3) shows that the IC and IR constraints of H-types are equivalent to $p_H R_H > I$, which holds by assumption. Thus, H^eL^{fs} is defined by two existence conditions. First, the condition

$$\chi_L^* \geq 0 \Leftrightarrow A \geq hI \quad (\text{C9})$$

must hold. Second, the L-type IC constraint (C2) must hold. By employing equations (C1) and (C7), equation (C2) can be rewritten as

$$A \geq \hat{A} + \frac{p_L \gamma}{\Delta p} = \bar{A}, \quad (\text{C10})$$

where the right-hand side equals equation (A10). Equations (C9) and (C10) show that H^eL^{fs} only exists in a non-wealth-constrained economy for $A \in [\bar{A}, I)$ (see Figure 2).

Appendix D: $H^{ef}L^e$ and $H^{efs}L^e$

We first prove that $H^{efs}L^e$ cannot exist. In $H^{efs}L^e$, $\mu_H \in (0, 1)$, $\mu_L = 1$, $\chi_H \in (0, 1)$, and $\chi_L = 0$. The five basic constraints in $H^{efs}L^e$ are

$$R_F = A, \tag{D1}$$

$$p_L(R_L - R_B) \geq R_F, \tag{D2}$$

$$p_H(R_H - R_B) = R_F, \tag{D3}$$

$$(1 - \mu_H - \chi_H)hA_{agg} = (1 - h + \mu_H h)(I_{aff} - A_{agg}), \tag{D4}$$

and

$$[h\mu_H p_H + (1 - h)p_L] R_B = R_F(1 - \mu_H - \chi_H)h. \tag{D5}$$

Equation (D1) shows both types of IR constraint, and equations (D2) and (D3) give the L-type and H-type IC constraints, respectively. Conditions (D4) and (D5) balance the supply and demand for funds.

A system consisting of equations (D1) and (D3)–(D5) determines the values of the endogenous variables R_F , R_B , χ_H , and μ_H . Because $R_F^* = A$ by equation (D1), equation (D3) gives R_B as

$$R_B^* = \frac{p_H R_H - A}{p_H}. \tag{D6}$$

By substituting equations (D1) and (D6) into equation (D5) and performing some intricate algebra, we can write equations (D4) and (D5) as

$$\mu_H^* = \frac{1 - h}{h} \left[1 - \frac{\Delta p(p_H R_H - A)}{p_H \gamma} \right] = \frac{1 - h}{h} \left(1 - \frac{R_B^* \Delta p}{\gamma} \right) \tag{D8}$$

and

$$\chi_H^* = \frac{1}{h} \left[1 - \frac{\Delta p(p_H R_H - A)I(1 - h)}{p_H \gamma A} \right] = \frac{1}{h} \left[1 - \frac{R_B^* \Delta p I(1 - h)}{\gamma A} \right] \tag{D9}$$

Equations (D8) and (D9) provide the following four equilibrium-existence conditions.

$$\mu_H^* \leq 1 \Leftrightarrow A \leq \frac{p_H [h\gamma + (1 - h)\lambda]}{\Delta p (1 - h)} \tag{D10}$$

$$\mu_H^* \geq 0 \Leftrightarrow A \geq \frac{p_H (I - p_L R_H)}{\Delta p} \tag{D11}$$

$$\chi_H^* \leq 1 \Leftrightarrow A \leq \frac{p_H R_H I \Delta p}{I \Delta p + p_H \gamma} = \frac{p_H R_H I \Delta p}{p_H^2 R_H - p_L I} \tag{D12}$$

$$\chi_H^* \geq 0 \Leftrightarrow A \geq \frac{p_H R_H I (1 - h) \Delta p}{I (1 - h) \Delta p + p_H \gamma} = \frac{p_H R_H I (1 - h) \Delta p}{p_H^2 R_H - I (p_L + h \Delta p)} \quad (D13)$$

With H-type IC and IR binding, the fifth equilibrium-existence condition comes from the L-type IC (D2), which implies that their IR is satisfied. Using (D1) and (D6), we see that the L type IC constraint (D2) is satisfied if

$$A \leq \hat{A}. \quad (D14)$$

Because the vertical line (D14) is smaller in value than the vertical line (D11), the equilibrium cannot exist for positive χ_H .

In contrast, $H^{ef}L^e$ exists. To see this, note first that in $H^{ef}L^e$, equation (D1) should be rewritten as an inequality, $R_F \geq A$. Then, we let $\chi_H = 0$ in equation (D4) in order to obtain

$$\mu_H^* = \frac{A - (1 - h)I}{hI}. \quad (D15)$$

Substituting equations (D15) and (D3) for equation (D5) and letting $\chi_H = 0$, we have

$$R_B^* = \frac{p_H R_H (I - A)}{I (p_L + h \Delta p)}. \quad (D16)$$

Inserting equation (D16) back into equation (D3) gives

$$R_F^* = \frac{p_H R_H [p_H A - \Delta p I (1 - h)]}{I (p_L + h \Delta p)}. \quad (D17)$$

From equation (D15) we see that $\mu_H^* < 1$ holds by our assumption that $A < I$. For $\mu_H^* \geq 0$, it is necessary to have

$$A \geq (1 - h)I. \quad (D18)$$

The H-type IR constraint is now $R_F \geq A$. Using equation (D17), this can be expressed as

$$A \geq \frac{p_H R_H I \Delta p (1 - h)}{I \Delta p (1 - h) + p_H \gamma} = \frac{p_H R_H I \Delta p (1 - h)}{p_H^2 R_H - I (p_L + h \Delta p)}. \quad (D19)$$

Similarly, using equations (D16) and (D17), the L-type IC constraint (D2) is given by

$$A \leq I - \frac{I \Delta W (p_L + h \Delta p)}{\Delta p p_H R_H} = \frac{I}{p_H R_H} [\hat{A} + \Delta W (1 - h)]. \quad (D20)$$

Conditions (D18)–(D20) characterize the existence of $H^{ef}L^e$. Equation (D18) is a downward-sloping $h = 1 - A/I$ -diagonal, which starts from the $(A = 0, h = 1)$ corner and ends in the $(A = I, h = 0)$ corner. The H-type IR constraint (D19) is a monotonically downward-sloping curve, which starts

from the ($A = 0, h = 1$) corner and cuts the $h = 1 - A/I$ -diagonal once after the vertical \hat{A} -line. The L-type IC constraint (D20) is a downward-sloping line, which begins from the vertical $\hat{A}I/p_H R_H$ -line (when $h = 1$), cutting the $h = 1 - A/I$ -diagonal after the vertical \hat{A} -line. The H-type IR and L-type IC constraints and the vertical \hat{A} -line cross at the same point at $h = h_2 \equiv 1 - \hat{A}\gamma/I\Delta W$, which is above the $h = 1 - A/I$ -diagonal. This makes equation (D18) redundant. In summary, $H^{ef}L^e$ exists above the H-type IR curve (D19) and below the L-type IC line (D20). This area is in the upper-left corner of the (A, h) -space where $A \in [0, \hat{A}]$ and $h \in [h_2, 1]$ (see Figure 2).

Appendix E: $H^{ef}L^{ef}$ and $H^{efs}L^{efs}$

We first prove that $H^{efs}L^{efs}$ cannot exist for a non-trivial set of parameters. In this equilibrium, all μ_H, μ_L, χ_H , and $\chi_L \in (0, 1)$. The IR and IC constraints of the agents bind. That is, it must hold that

$$R_F = A, \tag{E1}$$

$$p_L(R_L - R_B) = R_F, \tag{E2}$$

and

$$p_H(R_H - R_B) = R_F. \tag{E3}$$

Solving equations (E2) and (E3) for R_B gives

$$R_B^* = \frac{\Delta W}{\Delta p}. \tag{E4}$$

Thus, there is a unique value of

$$A = p_H \left(R_H - \frac{\Delta W}{\Delta p} \right) = p_L \left(R_L - \frac{\Delta W}{\Delta p} \right) = \hat{A}. \tag{E5}$$

for which this equilibrium can exist. This means that only $H^{ef}L^{ef}$ (where both μ_H and $\mu_L \in (0, 1)$ but $\chi_H = \chi_L = 0$) can exist for a non-trivial range of parameters.

$H^{ef}L^{ef}$ is constrained by the following five basic conditions.

$$R_F \geq A \tag{E6}$$

$$p_L(R_L - R_B) = R_F \tag{E7}$$

$$p_H(R_H - R_B) = R_F \tag{E8}$$

$$[(1 - \mu_H)h + (1 - \mu_L)(1 - h)] A_{agg} = [\mu_L(1 - h) + \mu_H h](I_{agg} - A_{agg}) \tag{E9}$$

$$[(1 - \mu_H)h + (1 - \mu_L)(1 - h)] R_F = [p_L \mu_L (1 - h) + p_H \mu_H h] R_B \quad (\text{E10})$$

Both types of IR constraint are given by equation (E6); equations (E7) and (E8) constitute L-type and H-type IC constraints, respectively. Conditions (E9) and (E10) balance the supply and demand for funds.

The system of equations (E7)–(E10) determines the values of the endogenous variables, R_F , R_B , μ_L , and μ_H . Solving equations (E7) and (E8) for R_B and R_F gives

$$R_B^* = \frac{\Delta W}{\Delta p} \quad (\text{E11})$$

and

$$R_F^* = p_H \left(R_H - \frac{\Delta W}{\Delta p} \right) = p_L \left(R_L - \frac{\Delta W}{\Delta p} \right) = \hat{A}. \quad (\text{E12})$$

Substituting equations (E11) and (E12) into equation (E10) and solving equations (E9) and (E10) for μ_L and μ_H yields

$$\mu_H^* = \frac{1}{h \Delta W} \left(\hat{A} - \frac{A p_L R_L}{I} \right) \quad (\text{E13})$$

and

$$\mu_L^* = \frac{1}{(1 - h) \Delta W} \left(\frac{A p_H R_H}{I} - \hat{A} \right). \quad (\text{E14})$$

Equations (E13) and (E14) yield four equilibrium-existence conditions.

$$\mu_H^* < 1 \Leftrightarrow A > \frac{I}{p_L R_L} (\hat{A} - h \Delta W) \quad (\text{E15})$$

$$\mu_H^* > 0 \Leftrightarrow A < \frac{I}{p_L R_L} \hat{A} \quad (\text{E16})$$

$$\mu_L^* < 1 \Leftrightarrow A < \frac{I}{p_H R_H} [\hat{A} + (1 - h) \Delta W] \quad (\text{E17})$$

$$\mu_L^* > 0 \Leftrightarrow A > \frac{I}{p_H R_H} \hat{A} \quad (\text{E18})$$

Equations (E6) and (E12) imply that the IR constraints of agents are satisfied if

$$A \leq \hat{A}. \quad (\text{E19})$$

This is the fifth equilibrium-existence condition. However, we see that if equation (E19) holds, equation (E16) also holds. The equilibrium is thus defined by equations (E15) and (E17)–(E19). Because equation (E17) is

identical to equation (D20), we know that it cuts the vertical \hat{A} -line at $h = h_2$, where $h_2 \equiv 1 - \hat{A}\gamma/I\Delta W$, as defined in Appendix D. This means that when h is large enough (i.e., $h \in [h_2, 1]$), the downward-sloping line (E17) and the vertical line (E18) are the binding constraints. For $h \in [h_3, h_2]$, where $h_3 \equiv \hat{A}/p_H R_H$, the binding constraints are the vertical lines (E18) and (E19). For $h \in [h_1, h_3]$, where $h_1 \equiv \hat{A}\lambda/I\Delta W$, as defined in Appendix A, the binding constraints are equations (E15) – which is identical to equation (15) – and (E19). For $h < h_1$, the equilibrium does not exist, because equation (E15) is violated.

In Figure 2, we illustrate how, in (A, h) -space, $H^{ef}L^{ef}$ exists in a parallelogram between the vertical lines (E18) and (E19) and the downward-sloping lines (E15) and (E17). This parallelogram exists for $A \in (\hat{A}I/p_H R_H, \hat{A})$ and $h \in [h_1, 1]$.

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